NEUTROSOPHIC CUBIC SET THEORY APPLIED TO UP-ALGEBRAS

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Thesis

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Neutrosophic Cubic Set Theory Applied to UP-Algebras

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Approved in partial fulfillment of the requirements for the

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บทคัดย่อ

เริ่มต้น เราแนะนำแนวคิดของพีชคณิตย่อยยูพีนิวโทรโซฟิก(พิเศษ) ตัวกรองยูพีใกล้นิวโทรโซฟิก(พิเศษ) ตัวกรองยูพีนิวโทรโซฟิก(พิเศษ) ไอดีลยูพีนิวโทรโซฟิก(พิเศษ) และไอดีลยูพีเข้มนิวโทรโซฟิก(พิเศษ) ของพีชคณิตยูพี และตรวจสอบคุณสมบัติต่าง ๆ ต่อจากนั้น เราแนะนำแนวคิดของพีชคณิตย่อยยูพีนิวโทรโซฟิกแบบช่วงค่า ตัวกรองยูพีใกล้นิวโทรโซฟิกแบบช่วงค่า ตัวกรองยูพีนิวโทรโซฟิกแบบช่วงค่า ไอดีลยูพีนิวโทรโซฟิกแบบช่วงค่า และไอดีลยูพีเข้มนิวโทรโซฟิกแบบช่วงค่าของพีชคณิตยูพี และพิสูจน์ผลลัพธ์บางอย่างที่สัมพันธ์กับแนวคิดก่อนหน้า จากสองแนวคิดข้างต้น เราแนะนำแนวคิดผสมของพีชคณิตย่อยยูพีกำลังสามนิวโทรโซฟิก ตัวกรองยูพีใกล้กำลังสามนิวโทรโซฟิก ตัวกรองยูพีกำลังสามนิวโทรโซฟิก ไอดีลยูพีกำลังสามนิวโทรโซฟิก และไอดีลยูพีเข้มกำลังสามนิวโทรโซฟิกของพีชคณิตยูพี เรายังกล่าวถึงความสัมพันธ์ระหว่างพีชคณิตย่อยยูพีกำลังสามนิวโทรโซฟิก (ตัวกรองยูพีใกล้กำลังสามนิวโทรโซฟิก ตัวกรองยูพีกำลังสามนิวโทรโซฟิก ไอดีลยูพีกำลังสามนิวโทรโซฟิก ไอดีลยูพีเข้มกำลังสามนิวโทรโซฟิก ตามลำดับ) และเซตย่อยระดับโดยวิถีทางของเซตนิวโทรโซฟิกแบบช่วงค่า และเซตนิวโทรโซฟิก มากกว่านั้น เราศึกษาภาพและภาพผกผันของพีชคณิตย่อยยูพีกำลังสามนิวโทรโซฟิก (ตัวกรองยูพีใกล้กำลังสามนิวโทรโซฟิก ตัวกรองยูพีกำลังสามนิวโทรโซฟิก ไอดีลยูพีกำลังสามนิวโทรโซฟิก ไอดีลยูพีเข้มกำลังสามนิวโทรโซฟิก ตามลำดับ) ภายใต้สาทิสสัณฐานยูพีบางอย่าง

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ABSTRACT

Initially, we introduce the concepts of (special) neutrosophic UP-subalgebras, (special) neutrosophic near UP-filters, (special) neutrosophic UP-filters, (special) neutrosophic UP-ideals, and (special) neutrosophic strong UP-ideals of UP-algebras, and investigate several properties. Next, we introduce the concepts of interval-valued neutrosophic UP-subalgebras, interval-valued neutrosophic near UP-filters, interval-valued neutrosophic UP-filters, interval-valued neutrosophic UP-ideals, and interval-valued neutrosophic strong UP-ideals of UP-algebras, and prove some results that are related to the previous concepts. From the two concepts above, we introduce the mixed concepts of neutrosophic cubic UP-subalgebras, neutrosophic cubic near UP-filters, neutrosophic cubic UP-filters, neutrosophic cubic UP-ideals, and neutrosophic cubic strong UP-ideals of UP-algebras. We also discuss the relationships among neutrosophic cubic UP-subalgebras (resp., neutrosophic cubic near UP-filters, neutrosophic cubic UP-filters, neutrosophic cubic UP-ideals, neutrosophic cubic strong UP-ideals) and their level subsets by means of interval-valued neutrosophic sets and neutrosophic sets. Moreover, we study the image and inverse image of neutrosophic cubic UP-subalgebras (resp., neutrosophic cubic near UP-filters, neutrosophic cubic UP-filters, neutrosophic cubic UP-ideals, neutrosophic cubic strong UP-ideals) under some UP-homomorphisms.

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CHAPTER I

INTRODUCTION

Among many algebraic structures, algebras of logic form important class of algebras. Examples of these are BCK-algebras [9], BCI-algebras [11], B-algebras [29], KU-algebras [30], UP-algebras [6] and others. They are strongly connected with logic. For example, BCI-algebras introduced by Iséki [11] in 1966 have connections with BCI-logic being the BCI-system in combinatory logic which has application in the language of functional programming. BCK and BCI-algebras are two classes of logical algebras. They were introduced by Imai and Iséki [9, 11] in 1966 and have been extensively investigated by many researchers. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras.

The branch of the logical algebra, a UP-algebra was introduced by Iampan [6], and it is known that the class of KU-algebras is a proper subclass of the class of UP-algebras. Later Somjanta et al. [38] studied fuzzy UP-subalgebras, fuzzy UP-ideals and fuzzy UP-filters of UP-algebras. Guntasow et al. [4] studied fuzzy translations of a fuzzy set in UP-algebras. Kesorn et al. [20] studied intuitionistic fuzzy sets in UP-algebras. Kaijae et al. [17] studied anti-fuzzy UP-ideals and anti-fuzzy UP-subalgebras. Tanamoon et al. [43] studied Q-fuzzy sets in UP-algebras. Sripaeng et al. [41] studied anti Q-fuzzy UP-ideals and anti Q-fuzzy UP-subalgebras of UP-algebras. Dokkhamdang et al. [3] studied generalized fuzzy sets in UP-algebras. Songsaeng and Iampan [39, 40] studied $\mathcal{N}$-fuzzy UP-algebras and fuzzy proper UP-filters of UP-algebras. Senapati et al. [36, 34] studies cubic set and interval-valued intuitionistic fuzzy structure in UP-algebras.

A fuzzy set f in a nonempty set S is a function from S to the closed interval $[0, 1]$. The concept of a fuzzy set in a nonempty set was first considered by Zadeh [46]. The fuzzy set theories developed by Zadeh and others have found many applications in the domain of mathematics and elsewhere. Zadeh [47] was introduced an interval-value fuzzy sets. An interval-valued fuzzy set is defined by an interval-valued membership function. The concept of neutrosophic set was introduced by Smarandache [37] in 1999. Wang et al. [45] introduced the concept of interval-valued neutrosophic sets in 2005. The interval-valued neutrosophic set is an instance of neutrosophic set which can be used in real scientific and engineering applications. Jun et al. [14] introduced the concept of interval-valued neutrosophic sets with applications in BCK/BCI-algebra, they also introduced the concept of interval-valued neutrosophic length of an interval-valued neutrosophic set, and investigate their properties and relations. In 2018-2019, Muhiuddin et al. [23, 24, 25, 26, 27, 28] applied the concept of neutrosophic sets to semigroups, BCK/BCI-algebras. The concept of neutrosophic $\mathcal{N}$-structures and their applications in semigroups was introduced Khan et al. [21] in 2017. Jun et al. [15] applied the concept of neutrosophic $\mathcal{N}$-structures to BCK/BCI-algebras in 2017.

A cubic set in a nonempty set is a structure using an interval-value fuzzy set and a fuzzy set was introduced by Jun et al. [13] in 2012. People find that cubic sets have board applications in computer science and soft engineering. Jun et al. [12] applied the concept of cubic sets to a subgroup in 2011. Senapati [35] introduced the concept of cubic subalgebras and cubic closed ideals of B-algebras in 2015. Senapati et al. [34] introduced the concept of cubic set structure applied in UP-algebras in 2018.

A neutrosophic cubic set which is the generalized form of fuzzy sets, cubic sets and neutrosophic sets and introduced by Jun et al. [16] in 2017. The concept of truth-internals (indeterminacy-internals, falsity-internals) and truth-externals (indeterminacy-externals, falsity-externals) were introduced and related properties were investigated. Iqbal et al. [10] introduced the concept of neutrosophic cubic subalgebras and neutrosophic cubic closed ideals of B-algebras in 2016. Relation among neutrosophic cubic algebra with neutrosophic cubic ideals and neutrosophic closed ideals of B-algebras were studied and some related properties were investigated.

CHAPTER II

PRELIMINARIES

In 1965, Zadeh [46] introduced the concept of a fuzzy set in a nonempty set as the following definition.

Definition 2.0.1 A *fuzzy set* (briefly, FS) in a nonempty set X (or a fuzzy subset of X) is defined to be a function $\lambda : X \to [0,1]$, where $[0,1]$ is the unit segment of the real line. Denote by $[0,1]^X$ the collection of all fuzzy sets in X. Define a binary relation $\leq$ on $[0,1]^X$ as follows:

$$(\forall \lambda, \mu \in [0,1]^X)(\lambda \leq \mu \Leftrightarrow (\forall x \in X)(\lambda(x) \leq \mu(x))). \tag{2.0.1}$$

Definition 2.0.2 [38] Let λ be a fuzzy set in a nonempty set X. The *complement of* λ, denoted by λ^C, is defined by

$$(\forall x \in X)(\lambda^C(x) = 1 - \lambda(x)). \tag{2.0.2}$$

Definition 2.0.3 [22] Let $\{\lambda_i \mid i \in J\}$ be a family of fuzzy sets in a nonempty set X. We define the *join* and the *meet* of $\{\lambda_i \mid i \in J\}$, denoted by $\vee_{i\in J}\lambda_i$ and $\wedge_{i\in J}\lambda_i$, respectively, as follows:

$$(\forall x \in X)((\vee_{i\in J}\lambda_i)(x) = \sup_{i\in J}\{\lambda_i(x)\}), \text{ and} \tag{2.0.3}$$

$$(\forall x \in X)((\wedge_{i\in J}\lambda_i)(x) = \inf_{i\in J}\{\lambda_i(x)\}). \tag{2.0.4}$$

In particular, if λ and μ be fuzzy sets in X, we have the join and meet of λ and μ as follows:

$$(\forall x \in X)((\lambda \vee \mu)(x) = \max\{\lambda(x), \mu(x)\}), \text{ and} \tag{2.0.5}$$

$$(\forall x \in X)((\lambda \wedge \mu)(x) = \min\{\lambda(x), \mu(x)\}), \tag{2.0.6}$$

respectively.

Lemma 2.0.4 *[44] Let $a, b, c \in \mathbb{R}$. Then the following statements hold:*

(1) $a - \min\{b, c\} = \max\{a - b, a - c\}$, *and*

(2) $a - \max\{b, c\} = \min\{a - b, a - c\}$.

The following lemma is easily proved.

Lemma 2.0.5 *Let f be a fuzzy set in a nonempty set X. Then the following statements hold:*

(1) $(\forall x, y, z \in X)(\overline{f}(x) \geq \min\{\overline{f}(y), \overline{f}(z)\} \Leftrightarrow f(x) \leq \max\{f(y), f(z)\})$,

(2) $(\forall x, y, z \in X)(\overline{f}(x) \leq \min\{\overline{f}(y), \overline{f}(z)\} \Leftrightarrow f(x) \geq \max\{f(y), f(z)\})$,

(3) $(\forall x, y, z \in X)(\overline{f}(x) \geq \max\{\overline{f}(y), \overline{f}(z)\} \Leftrightarrow f(x) \leq \min\{f(y), f(z)\})$, *and*

(4) $(\forall x, y, z \in X)(\overline{f}(x) \leq \max\{\overline{f}(y), \overline{f}(z)\} \Leftrightarrow f(x) \geq \min\{f(y), f(z)\})$.

An *interval number* we mean a close subinterval $\tilde{a} = [a^-, a^+]$ of $[0, 1]$, where $0 \leq a^- \leq a^+ \leq 1$. The interval number $\tilde{a} = [a^-, a^+]$ with $a^- = a^+$ is denoted by $\mathbf{a}$. Denote by $[[0, 1]]$ the set of all interval numbers.

Definition 2.0.6 [16] Let $\{\tilde{a}_i \mid i \in J\}$ be a family of interval numbers. We define the *refined infimum* and the *refined supremum* of $\{\tilde{a}_i \mid i \in J\}$, denoted by $\mathrm{rinf}_{i\in J}\tilde{a}_i$ and $\mathrm{rsup}_{i\in J}\tilde{a}_i$, respectively, as follows:

$$\mathrm{rinf}_{i\in J}\{\tilde{a}_i\} = [\inf_{i\in J}\{a_i^-\}, \inf_{i\in J}\{a_i^+\}], \text{ and} \tag{2.0.7}$$

$$\mathrm{rsup}_{i\in J}\{\tilde{a}_i\} = [\sup_{i\in J}\{a_i^-\}, \sup_{i\in J}\{a_i^+\}]. \tag{2.0.8}$$

In particular, if $\tilde{a}_1$ and $\tilde{a}_2$ are interval numbers, we define the *refined minimum* and the *refined maximum* of $\tilde{a}_1$ and $\tilde{a}_2$, denoted by $\text{rmin}\{\tilde{a}_1, \tilde{a}_2\}$ and $\text{rmax}\{\tilde{a}_1, \tilde{a}_2\}$, respectively, as follows:

$$\text{rmin}\{\tilde{a}_1, \tilde{a}_2\} = [\min\{a_1^-, a_2^-\}, \min\{a_1^+, a_2^+\}], \text{ and} \tag{2.0.9}$$

$$\text{rmax}\{\tilde{a}_1, \tilde{a}_2\} = [\max\{a_1^-, a_2^-\}, \max\{a_1^+, a_2^+\}]. \tag{2.0.10}$$

Definition 2.0.7 [16] Let $\tilde{a}_1$ and $\tilde{a}_2$ be interval numbers. We define the symbols "$\succeq$", "$\preceq$", "$=$" in case of $\tilde{a}_1$ and $\tilde{a}_2$ as follows:

$$\tilde{a}_1 \succeq \tilde{a}_2 \Leftrightarrow a_1^- \geq a_2^- \text{ and } a_1^+ \geq a_2^+, \tag{2.0.11}$$

and similarly we may have $\tilde{a}_1 \preceq \tilde{a}_2$ and $\tilde{a}_1 = \tilde{a}_2$. To say $\tilde{a}_1 \succ \tilde{a}_2$ (resp., $\tilde{a}_1 \prec \tilde{a}_2$) we mean $\tilde{a}_1 \succeq \tilde{a}_2$ and $\tilde{a}_1 \neq \tilde{a}_2$ (resp., $\tilde{a}_1 \preceq \tilde{a}_2$ and $\tilde{a}_1 \neq \tilde{a}_2$).

Definition 2.0.8 [47] Let $\tilde{a}$ be an interval number. The *complement of* $\tilde{a}$, denoted by $\tilde{a}^C$, is defined by the interval number

$$\tilde{a}^C = [1 - a^+, 1 - a^-]. \tag{2.0.12}$$

In the $[[0,1]]$, the following assertions are valid (see [42]).

$$(\forall \tilde{a} \in [[0,1]])(\tilde{a} \succeq \tilde{a}), \tag{2.0.13}$$

$$(\forall \tilde{a} \in [[0,1]])((\tilde{a}^C)^C = \tilde{a}), \tag{2.0.14}$$

$$(\forall \tilde{a} \in [[0,1]])(\text{rmax}\{\tilde{a}, \tilde{a}\} = \tilde{a} \text{ and } \text{rmin}\{\tilde{a}, \tilde{a}\} = \tilde{a}), \tag{2.0.15}$$

$$(\forall \tilde{a}_1, \tilde{a}_2 \in [[0,1]])(\text{rmax}\{\tilde{a}_1, \tilde{a}_2\} = \text{rmax}\{\tilde{a}_2, \tilde{a}_1\} \text{ and } \text{rmin}\{\tilde{a}_1, \tilde{a}_2\} = \text{rmin}\{\tilde{a}_2, \tilde{a}_1\}), \tag{2.0.16}$$

$$(\forall \tilde{a}_1, \tilde{a}_2 \in [[0,1]])(\text{rmax}\{\tilde{a}_1, \tilde{a}_2\} \succeq \tilde{a}_1 \text{ and } \tilde{a}_2 \succeq \text{rmin}\{\tilde{a}_1, \tilde{a}_2\}), \tag{2.0.17}$$

$$(\forall \tilde{a}_1, \tilde{a}_2 \in [[0,1]])(\tilde{a}_1 \succeq \tilde{a}_2 \Leftrightarrow \tilde{a}_1^C \preceq \tilde{a}_2^C), \tag{2.0.18}$$

$$(\forall \tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4 \in [[0,1]])(\tilde{a}_1 \succeq \tilde{a}_2, \tilde{a}_3 \succeq \tilde{a}_4 \Rightarrow \text{rmin}\{\tilde{a}_1, \tilde{a}_3\} \succeq \text{rmin}\{\tilde{a}_2, \tilde{a}_4\}), \tag{2.0.19}$$

$$(\forall \tilde{a}_1, \tilde{a}_2, \tilde{a}_3 \in [[0,1]])(\tilde{a}_1 \succeq \tilde{a}_2, \tilde{a}_3 \succeq \tilde{a}_2 \Leftrightarrow \text{rmin}\{\tilde{a}_1, \tilde{a}_3\} \succeq \tilde{a}_2), \tag{2.0.20}$$

$$(\forall \tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4 \in [[0,1]])(\tilde{a}_1 \succeq \tilde{a}_2, \tilde{a}_3 \succeq \tilde{a}_4 \Rightarrow \text{rmax}\{\tilde{a}_1, \tilde{a}_3\} \succeq \text{rmax}\{\tilde{a}_2, \tilde{a}_4\}), \tag{2.0.21}$$

$$(\forall \tilde{a}_1, \tilde{a}_2, \tilde{a}_3 \in [[0,1]])(\tilde{a}_2 \succeq \tilde{a}_1, \tilde{a}_2 \succeq \tilde{a}_3 \Leftrightarrow \tilde{a}_2 \succeq \text{rmax}\{\tilde{a}_1, \tilde{a}_3\}), \tag{2.0.22}$$

$$(\forall \tilde{a}_1, \tilde{a}_2 \in [[0,1]])(\tilde{a}_1 \succeq \tilde{a}_2 \Leftrightarrow \text{rmin}\{\tilde{a}_1, \tilde{a}_2\} = \tilde{a}_2), \tag{2.0.23}$$

$$(\forall \tilde{a}_1, \tilde{a}_2 \in [[0,1]])(\tilde{a}_1 \succeq \tilde{a}_2 \Leftrightarrow \text{rmax}\{\tilde{a}_1, \tilde{a}_2\} = \tilde{a}_1), \tag{2.0.24}$$

$$(\forall \tilde{a}_1, \tilde{a}_2 \in [[0,1]])(\text{rmin}\{\tilde{a}_1^C, \tilde{a}_2^C\} = \text{rmax}\{\tilde{a}_1, \tilde{a}_2\}^C), \tag{2.0.25}$$

$$(\forall \tilde{a}_1, \tilde{a}_2 \in [[0,1]])(\text{rmax}\{\tilde{a}_1^C, \tilde{a}_2^C\} = \text{rmin}\{\tilde{a}_1, \tilde{a}_2\}^C), \tag{2.0.26}$$

$$(\forall \tilde{a}_1, \tilde{a}_2, \tilde{a}_3 \in [[0,1]])(\tilde{a}_1 \preceq \text{rmax}\{\tilde{a}_2, \tilde{a}_3\} \Leftrightarrow \tilde{a}_1^C \succeq \text{rmin}\{\tilde{a}_2^C, \tilde{a}_3^C\}), \tag{2.0.27}$$

$$(\forall \tilde{a}_1, \tilde{a}_2, \tilde{a}_3 \in [[0,1]])(\tilde{a}_1 \succeq \text{rmax}\{\tilde{a}_2, \tilde{a}_3\} \Leftrightarrow \tilde{a}_1^C \preceq \text{rmin}\{\tilde{a}_2^C, \tilde{a}_3^C\}), \tag{2.0.28}$$

$$(\forall \tilde{a}_1, \tilde{a}_2, \tilde{a}_3 \in [[0,1]])(\tilde{a}_1 \preceq \text{rmin}\{\tilde{a}_2, \tilde{a}_3\} \Leftrightarrow \tilde{a}_1^C \succeq \text{rmax}\{\tilde{a}_2^C, \tilde{a}_3^C\}), \text{ and} \tag{2.0.29}$$

$$(\forall \tilde{a}_1, \tilde{a}_2, \tilde{a}_3 \in [[0,1]])(\tilde{a}_1 \succeq \text{rmin}\{\tilde{a}_2, \tilde{a}_3\} \Leftrightarrow \tilde{a}_1^C \preceq \text{rmax}\{\tilde{a}_2^C, \tilde{a}_3^C\}). \tag{2.0.30}$$

In 1975, Zadeh [47] introduced the concept of an interval-valued fuzzy set in a nonempty set as the following definition.

Definition 2.0.9 An *interval-valued fuzzy set* (briefly, an IVFS) in a nonempty set X is an arbitrary function $A : X \to [[0,1]]$. Let $IVFS(X)$ stands for the set of all IVFS in X. For every $A \in IVFS(X)$ and $x \in X, A(x) = [A^-(x), A^+(x)]$ is called the *degree of membership* of an element x to A, where A^-, A^+ are fuzzy sets in X which are called a *lower fuzzy set* and an *upper fuzzy set* in X, respectively. For simplicity, we denote $A = [A^-, A^+]$.

Definition 2.0.10 [47] Let A be an interval-valued fuzzy set in a nonempty set X. The *complement of* A, denoted by A^C, is defined as follows: $A^C(x) = A(x)^C$

for all $x \in X$, that is,

$$(\forall x \in X)(A^C(x) = [1 - A^+(x), 1 - A^-(x)]). \tag{2.0.31}$$

We note that $A^{C^-}(x) = 1 - A^+(x)$ and $A^{C^+}(x) = 1 - A^-(x)$ for all $x \in X$.

Definition 2.0.11 [16] Let A and B be interval-valued fuzzy sets in a nonempty set X. We define the symbols "$\subseteq$", "$\supseteq$", "$=$" in case of A and B as follows:

$$(\forall x \in X)(A \subseteq B \Leftrightarrow A(x) \preceq B(x)), \tag{2.0.32}$$

and similarly we may have $A \supseteq B$ and $A = B$.

Definition 2.0.12 [47] Let $\{A_i \mid i \in J\}$ be a family of interval-valued fuzzy sets in a nonempty set X. We define the *intersection* and the *union* of $\{A_i \mid i \in J\}$, denoted by $\cap_{i\in J} A_i$ and $\cup_{i\in J} A_i$, respectively, as follows:

$$(\forall x \in X)((\cap_{i\in J} A_i)(x) = \text{rinf}_{i\in J}\{A_i(x)\}), \text{ and} \tag{2.0.33}$$

$$(\forall x \in X)((\cup_{i\in J} A_i)(x) = \text{rsup}_{i\in J}\{A_i(x)\}). \tag{2.0.34}$$

We note that

$$(\forall x \in X)((\cap_{i\in J} A_i)^-(x) = (\wedge_{i\in J} A_i^-)(x) = \inf_{i\in J}\{A_i^-(x)\})$$

and

$$(\forall x \in X)((\cap_{i\in J} A_i)^+(x) = (\wedge_{i\in J} A_i^+)(x) = \inf_{i\in J}\{A_i^+(x)\}).$$

Similarly,

$$(\forall x \in X)((\cup_{i\in J} A_i)^-(x) = (\vee_{i\in J} A_i^-)(x) = \sup_{i\in J}\{A_i^-(x)\})$$

and

$$(\forall x \in X)((\cup_{i\in J} A_i)^+(x) = (\vee_{i\in J} A_i^+)(x) = \sup_{i\in J}\{A_i^+(x)\}).$$

In particular, if A_1 and A_2 are interval-valued fuzzy sets in X, we have the intersection and the union of A_1 and A_2 as follows:

$$(\forall x \in X)((A_1 \cap A_2)(x) = \text{rmin}\{A_1(x), A_2(x)\}), \text{ and} \tag{2.0.35}$$

$$(\forall x \in X)((A_1 \cup A_2)(x) = \text{rmax}\{A_1(x), A_2(x)\}). \tag{2.0.36}$$

In 1999, Smarandache [37] introduced the concept of a neutrosophic set in a nonempty set as the following definition.

Definition 2.0.13 A *neutrosophic set* (briefly, NS) in a nonempty set X is a structure of the form:

$$\Lambda = \{(x, \lambda_T(x), \lambda_I(x), \lambda_F(x)) \mid x \in X\}, \tag{2.0.37}$$

where $\lambda_T : X \to [0,1]$ is a *truth membership function*, $\lambda_I : X \to [0,1]$ is an *indeterminate membership function*, and $\lambda_F : X \to [0,1]$ is a *false membership function*. For our convenience, we will denote a NS as $\Lambda = (X, \lambda_T, \lambda_I, \lambda_F) = (X, \lambda_{T,I,F}) = \{(x, \lambda_T(x), \lambda_I(x), \lambda_F(x)) \mid x \in X\}$.

Definition 2.0.14 [37] Let Λ be a NS in a nonempty set X. The NS $\overline{\Lambda} = (X, \overline{\lambda}_{T,I,F})$ in X defined by

$$(\forall x \in X)\begin{pmatrix} \overline{\lambda}_T(x) = 1 - \lambda_T(x) \\ \overline{\lambda}_I(x) = 1 - \lambda_I(x) \\ \overline{\lambda}_F(x) = 1 - \lambda_F(x) \end{pmatrix}$$

is called the *complement* of Λ in X.

Remark 2.0.15 For all NS Λ in a nonempty set X, we have $\Lambda = \overline{\overline{\Lambda}}$.

In 2005, Wang et al. [45] introduced the concept of an interval-valued neutrosophic set in a nonempty set as the following definition.

Definition 2.0.16 An *interval-valued neutrosophic set* (briefly, IVNS) in a nonempty set X is a structure of the form:

$$\mathbf{A} := \{(x, A_T(x), A_I(x), A_F(x)) \mid x \in X\}, \qquad (2.0.38)$$

where A_T, A_I and A_F are interval-valued fuzzy sets in X, which are called an *interval truth membership function*, an *interval indeterminacy membership function* and an *interval falsity membership function*, respectively. For our convenience, we will denote a IVNS as

$$\mathbf{A} = (X, A_T, A_I, A_F) = (X, A_{T,I,F}) = \{(x, A_T(x), A_I(x), A_F(x)) \mid x \in X\}.$$

In 2012, Jun et al. [13] introduced the concept of a cubic set in a nonempty set as the following definition.

Definition 2.0.17 A *cubic set* (briefly, CS) in a nonempty set X is a structure of the form:

$$\mathbf{C} = \{(x, A(x), \lambda(x)) \mid x \in X\}, \qquad (2.0.39)$$

where A is an interval-valued fuzzy set in X and λ is a fuzzy set in X. For our convenience, we will denote a CS as

$$\mathbf{C} = (X, A, \lambda) = \{(x, A(x), \lambda(x)) \mid x \in X\}.$$

In 2017, Jun et al. [16] introduced the concept of a neutrosophic cubic set in a nonempty set as the following definition.

Definition 2.0.18 A *neutrosophic cubic set* in a nonempty set X is a pair

$\mathcal{C} = (\mathbf{A}, \Lambda)$, where $\mathbf{A} = \{(x, A_T(x), A_I(x), A_F(x)) \mid x \in X\}$ is an interval-valued neutrosophic set in X and $\Lambda = \{(x, \lambda_T(x), \lambda_I(x), \lambda_F(x)) \mid x \in X\}$ is a neutrosophic set in X.

For our convenience, we will denote neutrosophic cubic set as

$$\mathcal{C} = (A_{T,I,F}, \lambda_{T,I,F}) = \{(x, A_{T,I,F}(x), \lambda_{T,I,F}(x)) \mid x \in X\}.$$

CHAPTER III

BASIC RESULTS ON UP-ALGEBRAS

Two important classes of logical algebras, KU-algebras and UP-algebras were introduced by Prabpayak and Leerawat [30] in 2009, and Iampan [6] in 2017, respectively. Now, we recall the definitions of KU-algebras and UP-algebras as the following.

Definition 3.0.1 An algebra $X = (X, \cdot, 0)$ of type $(2, 0)$ is called a *KU-algebra*, where X is a nonempty set, $\cdot$ is a binary operation on X, and 0 is a fixed element of X (i.e., a nullary operation) if it satisfies the following axioms:

(KU-1) $(\forall x, y, z \in X)((y \cdot x) \cdot ((x \cdot z) \cdot (y \cdot z)) = 0)$,

(KU-2) $(\forall x \in X)(0 \cdot x = x)$,

(KU-3) $(\forall x \in X)(x \cdot 0 = 0)$, and

(KU-4) $(\forall x, y \in X)(x \cdot y = 0, y \cdot x = 0 \Rightarrow x = y)$.

Definition 3.0.2 An algebra $X = (X, \cdot, 0)$ of type $(2, 0)$ is called a *UP-algebra*, where X is a nonempty set, $\cdot$ is a binary operation on X, and 0 is a fixed element of X (i.e., a nullary operation) if it satisfies the following axioms:

(UP-1) $(\forall x, y, z \in X)((y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0)$,

(UP-2) $(\forall x \in X)(0 \cdot x = x)$,

(UP-3) $(\forall x \in X)(x \cdot 0 = 0)$, and

(UP-4) $(\forall x, y \in X)(x \cdot y = 0, y \cdot x = 0 \Rightarrow x = y)$.

From [6], we know that the concept of UP-algebras is a generalization of KU-algebras.

From [6], the binary relation $\leq$ on a UP-algebra $X = (X, \cdot, 0)$ is defined as follows:

$$(\forall x, y \in X)(x \leq y \Leftrightarrow x \cdot y = 0).$$

Example 3.0.3 [33] Let X be a universal set and let $\Omega \in \mathcal{P}(X)$ where $\mathcal{P}(X)$ means the power set of X. Let $\mathcal{P}_\Omega(X) = \{A \in \mathcal{P}(X) \mid \Omega \subseteq A\}$. Define a binary operation $\cdot$ on $\mathcal{P}_\Omega(X)$ by putting $A \cdot B = B \cap (A^C \cup \Omega)$ for all $A, B \in \mathcal{P}_\Omega(X)$ where A^C means the complement of a subset A. Then $(\mathcal{P}_\Omega(X), \cdot, \Omega)$ is a UP-algebra and we shall call it the *generalized power UP-algebra of type 1 with respect to* Ω. Let $\mathcal{P}^\Omega(X) = \{A \in \mathcal{P}(X) \mid A \subseteq \Omega\}$. Define a binary operation $*$ on $\mathcal{P}^\Omega(X)$ by putting $A * B = B \cup (A^C \cap \Omega)$ for all $A, B \in \mathcal{P}^\Omega(X)$. Then $(\mathcal{P}^\Omega(X), *, \Omega)$ is a UP-algebra and we shall call it the *generalized power UP-algebra of type 2 with respect to* Ω. In particular, $(\mathcal{P}(X), \cdot, \emptyset)$ is a UP-algebra and we shall call it the *power UP-algebra of type 1*, and $(\mathcal{P}(X), *, X)$ is a UP-algebra and we shall call it the *power UP-algebra of type 2*.

Example 3.0.4 [3] Let $\mathbb{N}$ be the set of all natural numbers with two binary operations $\circ$ and $\bullet$ defined by

$$(\forall x, y \in \mathbb{N}) \left(x \circ y = \begin{cases} y & \text{if } x < y, \\ 0 & \text{otherwise} \end{cases} \right)$$

and

$$(\forall x, y \in \mathbb{N}) \left(x \bullet y = \begin{cases} y & \text{if } x > y \text{ or } x = 0, \\ 0 & \text{otherwise} \end{cases} \right).$$

Then $(\mathbb{N}, \circ, 0)$ and $(\mathbb{N}, \bullet, 0)$ are UP-algebras.

For more examples of UP-algebras, see [1, 2, 7, 32, 33, 34, 36].

In a UP-algebra $X = (X, \cdot, 0)$, the following assertions are valid (see

[6, 7]).

$$(\forall x \in X)(x \cdot x = 0), \tag{3.0.1}$$

$$(\forall x, y, z \in X)(x \cdot y = 0, y \cdot z = 0 \Rightarrow x \cdot z = 0), \tag{3.0.2}$$

$$(\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow (z \cdot x) \cdot (z \cdot y) = 0), \tag{3.0.3}$$

$$(\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow (y \cdot z) \cdot (x \cdot z) = 0), \tag{3.0.4}$$

$$(\forall x, y \in X)(x \cdot (y \cdot x) = 0), \tag{3.0.5}$$

$$(\forall x, y \in X)((y \cdot x) \cdot x = 0 \Leftrightarrow x = y \cdot x), \tag{3.0.6}$$

$$(\forall x, y \in X)(x \cdot (y \cdot y) = 0), \tag{3.0.7}$$

$$(\forall a, x, y, z \in X)((x \cdot (y \cdot z)) \cdot (x \cdot ((a \cdot y) \cdot (a \cdot z))) = 0), \tag{3.0.8}$$

$$(\forall a, x, y, z \in X)((((a \cdot x) \cdot (a \cdot y)) \cdot z) \cdot ((x \cdot y) \cdot z) = 0), \tag{3.0.9}$$

$$(\forall x, y, z \in X)(((x \cdot y) \cdot z) \cdot (y \cdot z) = 0), \tag{3.0.10}$$

$$(\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow x \cdot (z \cdot y) = 0), \tag{3.0.11}$$

$$(\forall x, y, z \in X)(((x \cdot y) \cdot z) \cdot (x \cdot (y \cdot z)) = 0), \text{ and} \tag{3.0.12}$$

$$(\forall a, x, y, z \in X)(((x \cdot y) \cdot z) \cdot (y \cdot (a \cdot z)) = 0). \tag{3.0.13}$$

In UP-algebras, 5 types of special subsets are defined as follows.

Definition 3.0.5 [4, 5, 6, 38] A nonempty subset S of a UP-algebra $X = (X, \cdot, 0)$ is called

(1) a *UP-subalgebra* of X if $(\forall x, y \in S)(x \cdot y \in S)$.

(2) a *near UP-filter* of X if

  (i) the constant 0 of X is in S, and

  (ii) $(\forall x, y \in X)(y \in S \Rightarrow x \cdot y \in S)$.

(3) a *UP-filter* of X if

(i) the constant 0 of X is in S, and

(ii) $(\forall x, y \in X)(x \cdot y \in S, x \in S \Rightarrow y \in S)$.

(4) a *UP-ideal* of X if

(i) the constant 0 of X is in S, and

(ii) $(\forall x, y, z \in X)(x \cdot (y \cdot z) \in S, y \in S \Rightarrow x \cdot z \in S)$.

(5) a *strong UP-ideal* (renamed from a strongly UP-ideal) of X if

(i) the constant 0 of X is in S, and

(ii) $(\forall x, y, z \in X)((z \cdot y) \cdot (z \cdot x) \in S, y \in S \Rightarrow x \in S)$.

Guntasow et al. [4] and Iampan [5] proved that the concept of UP-subalgebras is a generalization of near UP-filters, near UP-filters is a generalization of UP-filters, UP-filters is a generalization of UP-ideals, and UP-ideals is a generalization of strong UP-ideals. Furthermore, they proved that the only strong UP-ideal of a UP-algebra X is X.

Theorem 3.0.6 *[4, 6, 31] Let $\mathscr{F}$ be a nonempty family of UP-subalgebras (resp., near UP-filters, UP-filters, UP-ideals, strong UP-ideals) of a UP-algebra $X = (X, \cdot, 0)$. Then $\bigcap \mathscr{F}$ is a UP-subalgebra (resp., near UP-filter, UP-filter, UP-ideal, strong UP-ideal) of X.*

Definition 3.0.7 [8, 7] Let $(X, \cdot, 0)$ and $(X', \cdot', 0')$ be UP-algebras. A mapping f from X to X' is called a *UP-homomorphism* if

$$f(x \cdot y) = f(x) \cdot' f(y) \quad \text{for all} \quad x, y \in X.$$

A UP-homomorphism $f : X \to X'$ is called a

(1) *UP-endomorphism* of X if $X' = X$,

(2) *UP-epimorphism* if f is surjective,

(3) *UP-monomorphism* if f is injective, and

(4) *UP-isomorphism* if f is bijective. Moreover, we say X is UP-isomorphic to X', symbolically, $X \cong X'$, if there is a UP-isomorphism from X to X'.

Theorem 3.0.8 *[8] Let $(X, \cdot, 0_X)$ and $(Y, *, 0_Y)$ be UP-algebras and let $f : X \to Y$ be a UP-homomorphism. Then the following statements hold:*

(1) $f(0_X) = 0_Y$,

(2) *for any $x, y \in X$, if $x \leq y$, then $f(x) \leq f(y)$.*

CHAPTER IV

MAIN RESULTS

4.1 Neutrosophic sets in UP-algebras

In this section, we introduce the concepts of neutrosophic UP-subalgebras, neutrosophic near UP-filters, neutrosophic UP-filters, neutrosophic UP-ideals, and neutrosophic strong UP-ideals of UP-algebras, provide the necessary examples, investigate their properties, and prove their generalizations.

From now on, unless another thing is stated, we take $X = (X, \cdot, 0)$ as a UP-algebra.

Definition 4.1.1 A NS Λ in X is called a *neutrosophic UP-subalgebra* of X if it satisfies the following conditions:

$$(\forall x, y \in X)(\lambda_T(x \cdot y) \geq \min\{\lambda_T(x), \lambda_T(y)\}), \tag{4.1.1}$$

$$(\forall x, y \in X)(\lambda_I(x \cdot y) \leq \max\{\lambda_I(x), \lambda_I(y)\}), \text{ and} \tag{4.1.2}$$

$$(\forall x, y \in X)(\lambda_F(x \cdot y) \geq \min\{\lambda_F(x), \lambda_F(y)\}). \tag{4.1.3}$$

Example 4.1.2 Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	2	4
2	0	0	0	2	4
3	0	0	0	0	4
4	0	1	2	3	0

We define a NS Λ in X as follows:

$$\lambda_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.9 & 0.7 & 0.5 & 0.3 & 0.3 \end{pmatrix}, \lambda_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 0.8 & 0.4 & 0.2 & 0.4 \end{pmatrix}, \text{ and}$$

$$\lambda_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 0.6 & 0.8 & 0.3 & 0.2 \end{pmatrix}.$$

Hence, Λ is a neutrosophic UP-subalgebra of X.

Definition 4.1.3 A NS Λ in X is called a *neutrosophic near UP-filter* of X if it satisfies the following conditions:

$$(\forall x \in X)(\lambda_T(0) \geq \lambda_T(x)), \tag{4.1.4}$$

$$(\forall x \in X)(\lambda_I(0) \leq \lambda_I(x)), \tag{4.1.5}$$

$$(\forall x \in X)(\lambda_F(0) \geq \lambda_F(x)), \tag{4.1.6}$$

$$(\forall x, y \in X)(\lambda_T(x \cdot y) \geq \lambda_T(y)), \tag{4.1.7}$$

$$(\forall x, y \in X)(\lambda_I(x \cdot y) \leq \lambda_I(y)), \text{ and} \tag{4.1.8}$$

$$(\forall x, y \in X)(\lambda_F(x \cdot y) \geq \lambda_F(y)). \tag{4.1.9}$$

Example 4.1.4 Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

·	0	1	2	3	4
0	0	1	2	3	4
1	0	0	1	2	4
2	0	0	0	1	4
3	0	0	0	0	4
4	0	1	2	3	0

We define a NS Λ in X as follows:

$$\lambda_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 0.7 & 0.5 & 0.4 & 0.8 \end{pmatrix}, \lambda_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.1 & 0.2 & 0.3 & 0.7 & 0.6 \end{pmatrix}, \text{ and}$$

$$\lambda_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.9 & 0.8 & 0.4 & 0.3 & 0.5 \end{pmatrix}.$$

Hence, Λ is a neutrosophic near UP-filter of X.

Definition 4.1.5 A NS Λ in X is called a *neutrosophic UP-filter* of X if it satisfies the following conditions: (4.1.4), (4.1.5), (4.1.6),

$$(\forall x, y \in X)(\lambda_T(y) \geq \min\{\lambda_T(x \cdot y), \lambda_T(x)\}), \tag{4.1.10}$$

$$(\forall x, y \in X)(\lambda_I(y) \leq \max\{\lambda_I(x \cdot y), \lambda_I(x)\}), \text{ and} \tag{4.1.11}$$

$$(\forall x, y \in X)(\lambda_F(y) \geq \min\{\lambda_F(x \cdot y), \lambda_F(x)\}). \tag{4.1.12}$$

Example 4.1.6 Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	0	0	3	3
3	0	1	2	0	3
4	0	1	2	0	0

We define a NS Λ in X as follows:

$$\lambda_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.9 & 0.4 & 0.3 & 0.1 & 0.1 \end{pmatrix}, \lambda_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.2 & 0.3 & 0.7 & 0.8 & 0.8 \end{pmatrix}, \text{ and}$$

$$\lambda_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.8 & 0.7 & 0.4 & 0.3 & 0.3 \end{pmatrix}.$$

Hence, Λ is a neutrosophic UP-filter of X.

Definition 4.1.7 A NS Λ in X is called a *neutrosophic UP-ideal* of X if it satisfies the following conditions: (4.1.4), (4.1.5), (4.1.6),

$$(\forall x, y, z \in X)(\lambda_T(x \cdot z) \geq \min\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\}), \tag{4.1.13}$$

$$(\forall x, y, z \in X)(\lambda_I(x \cdot z) \leq \max\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\}), \text{ and} \tag{4.1.14}$$

$$(\forall x, y, z \in X)(\lambda_F(x \cdot z) \geq \min\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}). \tag{4.1.15}$$

Example 4.1.8 Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	0	0	2	4
3	0	0	0	0	4
4	0	1	2	3	0

We define a NS Λ in X as follows:

$$\lambda_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 0.7 & 0.6 & 0.6 & 0.4 \end{pmatrix}, \lambda_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 0.3 & 0.5 & 0.5 & 0.7 \end{pmatrix}, \text{ and}$$

$$\lambda_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 0.8 & 0.7 & 0.7 & 0.5 \end{pmatrix}.$$

Hence, Λ is a neutrosophic UP-ideal of X.

Definition 4.1.9 A NS Λ in X is called a *neutrosophic strong UP-ideal* of X if it satisfies the following conditions: (4.1.4), (4.1.5), (4.1.6),

$$(\forall x, y, z \in X)(\lambda_T(x) \geq \min\{\lambda_T((z \cdot y) \cdot (z \cdot x)), \lambda_T(y)\}), \tag{4.1.16}$$

$$(\forall x, y, z \in X)(\lambda_I(x) \leq \max\{\lambda_I((z \cdot y) \cdot (z \cdot x)), \lambda_I(y)\}), \text{ and} \tag{4.1.17}$$

$$(\forall x, y, z \in X)(\lambda_F(x) \geq \min\{\lambda_F((z \cdot y) \cdot (z \cdot x)), \lambda_F(y)\}). \tag{4.1.18}$$

Example 4.1.10 Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	1	0	2	4
3	0	1	0	0	4
4	0	1	0	3	0

We define a NS Λ in X as follows:

$$(\forall x \in X)\begin{pmatrix} \lambda_T(x) = 1 \\ \lambda_I(x) = 0.2 \\ \lambda_F(x) = 0.8 \end{pmatrix}.$$

Hence, Λ is a neutrosophic strong UP-ideal of X.

Definition 4.1.11 A NS Λ in X is said to be *constant* if Λ is a constant function from X to $[0, 1]^3$. That is, λ_T, λ_I, and λ_F are constant functions from X to $[0, 1]$.

Theorem 4.1.12 *Every neutrosophic UP-subalgebra of X satisfies the conditions* (4.1.4)*,* (4.1.5)*, and* (4.1.6)*.*

Proof. Assume that Λ is a neutrosophic UP-subalgebra of X. Then for all $x \in X$,

$$\lambda_T(0) = \lambda_T(x \cdot x) \geq \min\{\lambda_T(x), \lambda_T(x)\} = \lambda_T(x), \qquad \text{((3.0.1) and (4.1.1))}$$

$$\lambda_I(0) = \lambda_I(x \cdot x) \leq \max\{\lambda_I(x), \lambda_I(x)\} = \lambda_I(x), \qquad \text{((3.0.1) and (4.1.2))}$$

$$\lambda_F(0) = \lambda_F(x \cdot x) \geq \min\{\lambda_F(x), \lambda_F(x)\} = \lambda_F(x). \qquad \text{((3.0.1) and (4.1.3))}$$

Hence, Λ satisfies the conditions (4.1.4), (4.1.5), and (4.1.6). □

Theorem 4.1.13 *A NS Λ in X is constant if and only if it is a neutrosophic strong UP-ideal of X.*

Proof. Assume that Λ is constant. Then for all $x \in X$, $\lambda_T(x) = \lambda_T(0), \lambda_I(x) = \lambda_I(0)$, and $\lambda_F(x) = \lambda_F(0)$ and so $\lambda_T(0) \geq \lambda_T(x), \lambda_I(0) \leq \lambda_I(x)$, and $\lambda_F(0) \geq \lambda_F(x)$. Next, for all $x, y, z \in X$,

$$\lambda_T(x) = \lambda_T(0) = \min\{\lambda_T(0), \lambda_T(0)\} = \min\{\lambda_T((z \cdot y) \cdot (z \cdot x)), \lambda_T(y)\},$$
$$\lambda_I(x) = \lambda_I(0) = \max\{\lambda_I(0), \lambda_I(0)\} = \max\{\lambda_I((z \cdot y) \cdot (z \cdot x)), \lambda_I(y)\},$$
$$\lambda_F(x) = \lambda_F(0) = \min\{\lambda_F(0), \lambda_F(0)\} = \min\{\lambda_F((z \cdot y) \cdot (z \cdot x)), \lambda_F(y)\}.$$

Hence, Λ is a neutrosophic strong UP-ideal of X.

Conversely, assume that Λ is a neutrosophic strong UP-ideal of X. For any $x \in X$, we have

$$\begin{aligned}
\lambda_T(x) &\geq \min\{\lambda_T((x \cdot 0) \cdot (x \cdot x)), \lambda_T(0)\} && \text{((4.1.16))}\\
&= \min\{\lambda_T(0 \cdot (x \cdot x)), \lambda_T(0)\} && \text{((UP-3))}\\
&= \min\{\lambda_T(x \cdot x), \lambda_T(0)\} && \text{((UP-2))}\\
&= \min\{\lambda_T(0), \lambda_T(0)\} && \text{((3.0.1))}\\
&= \lambda_T(0),\\
\lambda_I(x) &\leq \max\{\lambda_I((x \cdot 0) \cdot (x \cdot x)), \lambda_I(0)\} && \text{((4.1.17))}\\
&= \max\{\lambda_I(0 \cdot (x \cdot x)), \lambda_I(0)\} && \text{((UP-3))}\\
&= \max\{\lambda_I(x \cdot x), \lambda_I(0)\} && \text{((UP-2))}\\
&= \max\{\lambda_I(0), \lambda_I(0)\} && \text{((3.0.1))}
\end{aligned}$$

$$
\begin{aligned}
&= \lambda_I(0), \\
\lambda_F(x) &\geq \min\{\lambda_F((x \cdot 0) \cdot (x \cdot x)), \lambda_F(0)\} && ((4.1.18)) \\
&= \min\{\lambda_F(0 \cdot (x \cdot x)), \lambda_F(0)\} && ((\text{UP-3})) \\
&= \min\{\lambda_F(x \cdot x), \lambda_F(0)\} && ((\text{UP-2})) \\
&= \min\{\lambda_F(0), \lambda_F(0)\} && ((3.0.1)) \\
&= \lambda_F(0).
\end{aligned}
$$

Thus $\lambda_T(x) = \lambda_T(0), \lambda_I(x) = \lambda_I(0)$, and $\lambda_F(x) = \lambda_F(0)$ for all $x \in X$. Hence, Λ is constant. □

Theorem 4.1.14 *Every neutrosophic strong UP-ideal of X is a neutrosophic UP-ideal.*

Proof. Assume that Λ is a neutrosophic strong UP-ideal of X. Then Λ satisfies the conditions (4.1.4), (4.1.5), and (4.1.6). By Theorem 4.1.13, we have Λ is constant. Let $x, y, z \in X$. Then

$$
\begin{aligned}
\lambda_T(x \cdot z) = \lambda_T(y) &\geq \min\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\}, \\
\lambda_I(x \cdot z) = \lambda_I(y) &\leq \max\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\}, \\
\lambda_F(x \cdot z) = \lambda_F(y) &\geq \min\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}.
\end{aligned}
$$

Hence, Λ is a neutrosophic UP-ideal of X. □

The following example show that the converse of Theorem 4.1.14 is not true.

Example 4.1.15 From Example 4.1.8, we have Λ is a neutrosophic UP-ideal of X. Since Λ is not constant, it follows from Theorem 4.1.13 that it is not a neutrosophic strong UP-ideal of X.

Theorem 4.1.16 *Every neutrosophic UP-ideal of X is a neutrosophic UP-filter.*

Proof. Assume that Λ is a neutrosophic UP-ideal of X. Then Λ satisfies the conditions (4.1.4), (4.1.5), and (4.1.6). Next, let $x, y \in X$. Then

$$\begin{aligned}
\lambda_T(y) &= \lambda_T(0 \cdot y) && \text{((UP-2))}\\
&\geq \min\{\lambda_T(0 \cdot (x \cdot y)), \lambda_T(x)\} && \text{((4.1.13))}\\
&= \min\{\lambda_T(x \cdot y), \lambda_T(x)\}, && \text{((UP-2))}\\
\lambda_I(y) &= \lambda_I(0 \cdot y) && \text{((UP-2))}\\
&\leq \max\{\lambda_I(0 \cdot (x \cdot y)), \lambda_I(x)\} && \text{((4.1.14))}\\
&= \max\{\lambda_I(x \cdot y), \lambda_I(x)\}, && \text{((UP-2))}\\
\lambda_F(y) &= \lambda_F(0 \cdot y) && \text{((UP-2))}\\
&\geq \min\{\lambda_F(0 \cdot (x \cdot y)), \lambda_F(x)\} && \text{((4.1.15))}\\
&= \min\{\lambda_F(x \cdot y), \lambda_F(x)\}. && \text{((UP-2))}
\end{aligned}$$

Hence, Λ is a neutrosophic UP-filter of X. □

The following example show that the converse of Theorem 4.1.16 is not true.

Example 4.1.17 From Example 4.1.6, we have Λ is a neutrosophic UP-filter of X. Since $\lambda_F(3 \cdot 4) = 0.3 < 0.4 = \min\{\lambda_F(3 \cdot (2 \cdot 4)), \lambda_F(2)\}$, we have Λ is not a neutrosophic UP-ideal of X.

Theorem 4.1.18 *Every neutrosophic UP-filter of X is a neutrosophic near UP-filter.*

Proof. Assume that Λ is a neutrosophic UP-filter. Then Λ satisfies the conditions

(4.1.4), (4.1.5), and (4.1.6). Next, let $x, y \in X$. Then

$$\begin{aligned}
\lambda_T(x \cdot y) &\geq \min\{\lambda_T(y \cdot (x \cdot y)), \lambda_T(y)\} && ((4.1.10))\\
&= \min\{\lambda_T(0), \lambda_T(y)\} && ((3.0.5))\\
&= \lambda_T(y), && ((4.1.4))\\
\lambda_I(x \cdot y) &\leq \max\{\lambda_I(y \cdot (x \cdot y)), \lambda_I(y)\} && ((4.1.11))\\
&= \max\{\lambda_I(0), \lambda_I(y)\} && ((3.0.5))\\
&= \lambda_I(y), && ((4.1.5))\\
\lambda_F(x \cdot y) &\geq \min\{\lambda_F(y \cdot (x \cdot y)), \lambda_F(y)\} && ((4.1.12))\\
&= \min\{\lambda_F(0), \lambda_F(y)\} && ((3.0.5))\\
&= \lambda_F(y). && ((4.1.6))
\end{aligned}$$

Hence, Λ is a neutrosophic near UP-filter of X. □

The following example show that the converse of Theorem 4.1.18 is not true.

Example 4.1.19 From Example 4.1.4, we have Λ is a neutrosophic near UP-filter of X. Since $\lambda_I(3) = 0.7 > 0.3 = \max\{\lambda_I(2 \cdot 3), \lambda_I(2)\}$, we have Λ is not a neutrosophic UP-filter of X.

Theorem 4.1.20 *Every neutrosophic near UP-filter of X is a neutrosophic UP-subalgebra.*

Proof. Assume that Λ is a neutrosophic near UP-filter of X. Then for all $x, y \in X$

$$\begin{aligned}
\lambda_T(x \cdot y) \geq \lambda_T(y) \geq \min\{\lambda_T(x), \lambda_T(y)\}, && ((4.1.7))\\
\lambda_I(x \cdot y) \leq \lambda_I(y) \leq \max\{\lambda_I(x), \lambda_I(y)\}, && ((4.1.8))\\
\lambda_F(x \cdot y) \geq \lambda_F(y) \geq \min\{\lambda_F(x), \lambda_F(y)\}. && ((4.1.9))
\end{aligned}$$

Hence, Λ is a neutrosophic UP-subalgebra of X. □

The following example show that the converse of Theorem 4.1.20 is not true.

Example 4.1.21 From Example 4.1.2, we have Λ is a neutrosophic UP-subalgebra of X. Since $\lambda_I(2 \cdot 3) = 0.4 > 0.2 = \lambda_I(3)$, we have Λ is not a neutrosophic near UP-filter of X.

Theorem 4.1.22 *If Λ is a neutrosophic UP-subalgebra of X satisfying the following condition:*

$$(\forall x, y \in X)\left(x \cdot y \neq 0 \Rightarrow \begin{cases} \lambda_T(x) \geq \lambda_T(y) \\ \lambda_I(x) \leq \lambda_I(y) \\ \lambda_F(x) \geq \lambda_F(y) \end{cases} \right), \tag{4.1.19}$$

then Λ is a neutrosophic near UP-filter of X.

Proof. Assume that Λ is a neutrosophic UP-subalgebra of X satisfying the condition (4.1.19). By Theorem 4.1.12, we have Λ satisfies the conditions (4.1.4), (4.1.5), and (4.1.6). Next, let $x, y \in X$.

Case 1: $x \cdot y = 0$. Then

$$\lambda_T(x \cdot y) = \lambda_T(0) \geq \lambda_T(y), \qquad ((4.1.4))$$

$$\lambda_I(x \cdot y) = \lambda_I(0) \leq \lambda_I(y), \qquad ((4.1.5))$$

$$\lambda_F(x \cdot y) = \lambda_F(0) \geq \lambda_F(y). \qquad ((4.1.6))$$

Case 2: $x \cdot y \neq 0$. Then

$$\lambda_T(x \cdot y) \geq \min\{\lambda_T(x), \lambda_T(y)\} = \lambda_T(y), \qquad ((4.1.1) \text{ and } (4.1.19) \text{ for } \lambda_T)$$

$$\lambda_I(x \cdot y) \leq \max\{\lambda_I(x), \lambda_I(y)\} = \lambda_I(y), \qquad ((4.1.2) \text{ and } (4.1.19) \text{ for } \lambda_I)$$

$$\lambda_F(x \cdot y) \geq \min\{\lambda_F(x), \lambda_F(y)\} = \lambda_F(y). \qquad ((4.1.3) \text{ and } (4.1.19) \text{ for } \lambda_F)$$

Hence, Λ is a neutrosophic near UP-filter of X. □

Theorem 4.1.23 *If Λ is a neutrosophic near UP-filter of X satisfying the following condition:*

$$\lambda_T = \lambda_I = \lambda_F, \tag{4.1.20}$$

then Λ is a neutrosophic strong UP-ideal of X.

Proof. Assume that Λ is a neutrosophic near UP-filter of X satisfying the condition (4.1.20). Then Λ satisfies the conditions (4.1.4), (4.1.5), and (4.1.6). Let $x \in X$. Then

$$\lambda_T(0) \geq \lambda_T(x) = \lambda_I(x) \geq \lambda_I(0) = \lambda_T(0),$$

$$\lambda_I(0) \leq \lambda_I(x) = \lambda_T(x) \leq \lambda_T(0) = \lambda_I(0),$$

$$\lambda_F(0) \geq \lambda_F(x) = \lambda_I(x) \geq \lambda_I(0) = \lambda_F(0).$$

Thus $\lambda_T(0) = \lambda_T(x), \lambda_I(0) = \lambda_I(x)$, and $\lambda_F(0) = \lambda_F(x)$, that is, Λ is constant. By Theorem 4.1.13, we have Λ is a neutrosophic strong UP-ideal of X. □

Theorem 4.1.24 *If Λ is a neutrosophic UP-filter of X satisfying the following condition:*

$$(\forall x, y, z \in X)\begin{pmatrix} \lambda_T(y \cdot (x \cdot z)) = \lambda_T(x \cdot (y \cdot z)) \\ \lambda_I(y \cdot (x \cdot z)) = \lambda_I(x \cdot (y \cdot z)) \\ \lambda_F(y \cdot (x \cdot z)) = \lambda_F(x \cdot (y \cdot z)) \end{pmatrix} \tag{4.1.21}$$

then Λ is a neutrosophic UP-ideal of X.

Proof. Assume that Λ is a neutrosophic UP-filter of X satisfying the condition (4.1.21). Then Λ satisfies the conditions (4.1.4), (4.1.5), and (4.1.6). Next, let

$x, y, z \in X$. Then

$$\begin{aligned}
\lambda_T(x \cdot z) &\geq \min\{\lambda_T(y \cdot (x \cdot z)), \lambda_T(y)\} && ((4.1.10)) \\
&= \min\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\}, && ((4.1.21) \text{ for } \lambda_T) \\
\lambda_I(x \cdot z) &\leq \max\{\lambda_I(y \cdot (x \cdot z)), \lambda_I(y)\} && ((4.1.11)) \\
&= \max\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\}, && ((4.1.21) \text{ for } \lambda_I) \\
\lambda_F(x \cdot z) &\geq \min\{\lambda_F(y \cdot (x \cdot z)), \lambda_F(y)\} && ((4.1.12)) \\
&= \min\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}. && ((4.1.21) \text{ for } \lambda_F)
\end{aligned}$$

Hence, Λ is a neutrosophic UP-ideal of X. □

Theorem 4.1.25 *If Λ is a NS in X satisfying the following condition:*

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} \lambda_T(z) \geq \min\{\lambda_T(x), \lambda_T(y)\} \\ \lambda_I(z) \leq \max\{\lambda_I(x), \lambda_I(y)\} \\ \lambda_F(z) \geq \min\{\lambda_F(x), \lambda_F(y)\} \end{cases} \right), \tag{4.1.22}$$

then Λ is a neutrosophic UP-subalgebra of X.

Proof. Assume that Λ is a NS in X satisfying the condition (4.1.22). Let $x, y \in X$. By (3.0.1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \leq x \cdot y$. It follows from (4.1.22) that

$$\begin{aligned}
\lambda_T(x \cdot y) &\geq \min\{\lambda_T(x), \lambda_T(y)\}, \\
\lambda_I(x \cdot y) &\leq \max\{\lambda_I(x), \lambda_I(y)\}, \\
\lambda_F(x \cdot y) &\geq \min\{\lambda_F(x), \lambda_F(y)\}.
\end{aligned}$$

Hence, Λ is a neutrosophic UP-subalgebra of X. □

Theorem 4.1.26 *If Λ is a NS in X satisfying the following condition:*

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} \lambda_T(y) \geq \min\{\lambda_T(z), \lambda_T(x)\} \\ \lambda_I(y) \leq \max\{\lambda_I(z), \lambda_I(x)\} \\ \lambda_F(y) \geq \min\{\lambda_F(z), \lambda_F(x)\} \end{cases}\right), \tag{4.1.23}$$

then Λ is a neutrosophic UP-filter of X.

Proof. Assume that Λ is a NS in X satisfying the condition (4.1.23). Let $x \in X$. By (UP-3), we have $x \cdot (x \cdot 0) = 0$, that is, $x \leq x \cdot 0$. It follows from (4.1.23) that

$$\lambda_T(0) \geq \min\{\lambda_T(x), \lambda_T(x)\} = \lambda_T(x),$$
$$\lambda_I(0) \leq \max\{\lambda_I(x), \lambda_I(x)\} = \lambda_I(x),$$
$$\lambda_F(0) \geq \min\{\lambda_F(x), \lambda_F(x)\} = \lambda_F(x).$$

Next, let $x, y \in X$. By (3.0.1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \leq x \cdot y$. It follows from (4.1.23) that

$$\lambda_T(y) \geq \min\{\lambda_T(x \cdot y), \lambda_T(x)\},$$
$$\lambda_I(y) \leq \max\{\lambda_I(x \cdot y), \lambda_I(x)\},$$
$$\lambda_F(y) \geq \min\{\lambda_F(x \cdot y), \lambda_F(x)\}.$$

Hence, Λ is a neutrosophic UP-filter of X. □

Theorem 4.1.27 *If Λ is a NS in X satisfying the following condition:*

$$(\forall a, x, y, z \in X)\left(a \leq x \cdot (y \cdot z) \Rightarrow \begin{cases} \lambda_T(x \cdot z) \geq \min\{\lambda_T(a), \lambda_T(y)\} \\ \lambda_I(x \cdot z) \leq \max\{\lambda_I(a), \lambda_I(y)\} \\ \lambda_F(x \cdot z) \geq \min\{\lambda_F(a), \lambda_F(y)\} \end{cases}\right), \tag{4.1.24}$$

then Λ is a neutrosophic UP-ideal of X.

Proof. Assume that Λ is a NS in X satisfying the condition (4.1.24). Let $x \in X$. By (UP-3), we have $x \cdot (0 \cdot (x \cdot 0) = 0$, that is, $x \leq 0 \cdot (x \cdot 0)$. It follows from (4.1.24) that

$$\lambda_T(0) = \lambda_T(0 \cdot 0) \geq \min\{\lambda_T(x), \lambda_T(x)\} = \lambda_T(x), \qquad \text{((UP-2))}$$

$$\lambda_I(0) = \lambda_I(0 \cdot 0) \leq \max\{\lambda_I(x), \lambda_I(x)\} = \lambda_I(x), \qquad \text{((UP-2))}$$

$$\lambda_F(0) = \lambda_F(0 \cdot 0) \geq \min\{\lambda_F(x), \lambda_F(x)\} = \lambda_F(x). \qquad \text{((UP-2))}$$

Next, let $x, y, z \in X$. By (3.0.1), we have $(x \cdot (y \cdot z)) \cdot (x \cdot (y \cdot z)) = 0$, that is, $x \cdot (y \cdot z) \leq x \cdot (y \cdot z)$. It follows from (4.1.24) that

$$\lambda_T(x \cdot z) \geq \min\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\},$$

$$\lambda_I(x \cdot z) \leq \max\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\},$$

$$\lambda_F(x \cdot z) \geq \min\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}.$$

Hence, Λ is a neutrosophic UP-ideal of X. □

Theorem 4.1.28 *A NS Λ in X satisfies the following condition:*

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} \lambda_T(z) \geq \lambda_T(y) \\ \lambda_I(z) \leq \lambda_I(y) \\ \lambda_F(z) \geq \lambda_F(y) \end{cases} \right) \qquad (4.1.25)$$

if and only if Λ is a neutrosophic strong UP-ideal of X.

Proof. Assume that Λ is a NS in X satisfying the condition (4.1.25). Let $x, y \in X$. By (UP-3) and (3.0.1), we have $x \cdot 0 = 0$, that is, $x \leq 0 = y \cdot y$. It follows from (4.1.25) that $\lambda_T(x) \geq \lambda_T(y), \lambda_I(x) \leq \lambda_I(y)$, and $\lambda_F(x) \geq \lambda_F(y)$. Similarly,

$\lambda_T(y) \geq \lambda_T(x), \lambda_I(y) \leq \lambda_I(x)$, and $\lambda_F(y) \geq \lambda_F(x)$. Then $\lambda_T(x) = \lambda_T(y), \lambda_I(x) = \lambda_I(y)$, and $\lambda_F(x) = \lambda_F(y)$. Thus Λ is constant. By Theorem 4.1.13, we have Λ is a neutrosophic strong UP-ideal of X.

The converse follows from Theorem 4.1.13. □

Then, we have the diagram of generalization of NSs in UP-algebras as shown in Figure 4.1.

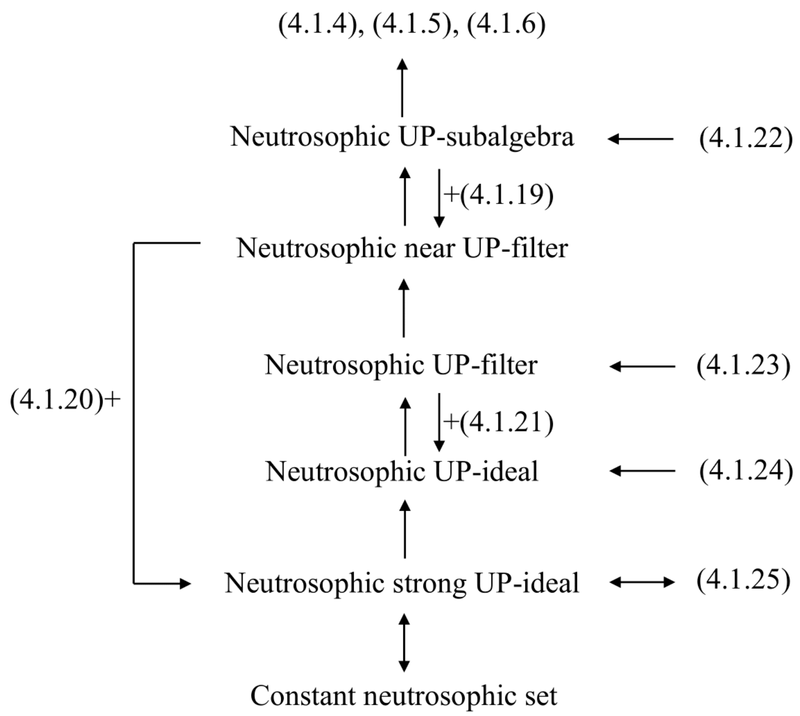

Figure 4.1: Neutrosophic sets in UP-algebras

For any fixed numbers $\alpha^+, \alpha^-, \beta^+, \beta^-, \gamma^+, \gamma^- \in [0,1]$ such that $\alpha^+ > \alpha^-, \beta^+ > \beta^-, \gamma^+ > \gamma^-$ and a nonempty subset G of X, the NS $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}] = (X, \lambda_T^G[^{\alpha^+}_{\alpha^-}], \lambda_I^G[^{\beta^-}_{\beta^+}], \lambda_F^G[^{\gamma^+}_{\gamma^-}])$ in X, where $\lambda_T^G[^{\alpha^+}_{\alpha^-}], \lambda_I^G[^{\beta^-}_{\beta^+}]$, and $\lambda_F^G[^{\gamma^+}_{\gamma^-}]$ are fuzzy sets in X which are given as follows:

$$\lambda_T^G[{}^{\alpha^+}_{\alpha^-}](x) = \begin{cases} \alpha^+ & \text{if } x \in G, \\ \alpha^- & \text{otherwise,} \end{cases}$$

$$\lambda_I^G[{}^{\beta^-}_{\beta^+}](x) = \begin{cases} \beta^- & \text{if } x \in G, \\ \beta^+ & \text{otherwise,} \end{cases}$$

$$\lambda_F^G[{}^{\gamma^+}_{\gamma^-}](x) = \begin{cases} \gamma^+ & \text{if } x \in G, \\ \gamma^- & \text{otherwise.} \end{cases}$$

Lemma 4.1.29 *If the constant* 0 *of* X *is in a nonempty subset* G *of* X*, then a NS* $\Lambda^G[{}^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ *in* X *satisfies the conditions* (4.1.4), (4.1.5), *and* (4.1.6).

Proof. If $0 \in G$, then $\lambda_T^G[{}^{\alpha^+}_{\alpha^-}](0) = \alpha^+, \lambda_I^G[{}^{\beta^-}_{\beta^+}](0) = \beta^-, \lambda_F^G[{}^{\gamma^+}_{\gamma^-}](0) = \gamma^+$. Thus

$$(\forall x \in X) \begin{pmatrix} \lambda_T^G[{}^{\alpha^+}_{\alpha^-}](0) = \alpha^+ \geq \lambda_T^G[{}^{\alpha^+}_{\alpha^-}](x) \\ \lambda_I^G[{}^{\beta^-}_{\beta^+}](0) = \beta^- \leq \lambda_I^G[{}^{\beta^-}_{\beta^+}](x) \\ \lambda_F^G[{}^{\gamma^+}_{\gamma^-}](0) = \gamma^+ \geq \lambda_F^G[{}^{\gamma^+}_{\gamma^-}](x) \end{pmatrix}.$$

Hence, $\Lambda^G[{}^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ satisfies the conditions (4.1.4), (4.1.5), and (4.1.6). □

Lemma 4.1.30 *If a NS* $\Lambda^G[{}^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ *in* X *satisfies the condition* (4.1.4) *(resp.,* (4.1.5), (4.1.6)*), then the constant* 0 *of* X *is in* G.

Proof. Assume that the NS $\Lambda^G[{}^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ in X satisfies the condition (4.1.4). Then $\lambda_T^G[{}^{\alpha^+}_{\alpha^-}](0) \geq \lambda_T^G[{}^{\alpha^+}_{\alpha^-}](x)$ for all $x \in X$. Since G is nonempty, there exists $g \in G$. Thus $\lambda_T^G[{}^{\alpha^+}_{\alpha^-}](g) = \alpha^+$ and so $\lambda_T^G[{}^{\alpha^+}_{\alpha^-}](0) \geq \lambda_T^G[{}^{\alpha^+}_{\alpha^-}](g) = \alpha^+ \geq \lambda_T^G[{}^{\alpha^+}_{\alpha^-}](0)$, that is, $\lambda_T^G[{}^{\alpha^+}_{\alpha^-}](0) = \alpha^+$. Hence, $0 \in G$. □

Theorem 4.1.31 *A NS* $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ *in* X *is a neutrosophic UP-subalgebra of* X *if and only if a nonempty subset* G *of* X *is a UP-subalgebra of* X*.*

Proof. Assume that $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a neutrosophic UP-subalgebra of X. Let $x, y \in G$. Then $\lambda^G_T[^{\alpha^+}_{\alpha^-}](x) = \alpha^+ = \lambda^G_T[^{\alpha^+}_{\alpha^-}](y)$. Thus

$$\lambda^G_T[^{\alpha^+}_{\alpha^-}](x \cdot y) \geq \min\{\lambda^G_T[^{\alpha^+}_{\alpha^-}](x), \lambda^G_T[^{\alpha^+}_{\alpha^-}](y)\} = \alpha^+ \geq \lambda^G_T[^{\alpha^+}_{\alpha^-}](x \cdot y) \qquad ((4.1.1))$$

and so $\lambda^G_T[^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^+$. Thus $x \cdot y \in G$. Hence, G is a UP-subalgebra of X.

Conversely, assume that G is a UP-subalgebra of X. Let $x, y \in X$.

Case 1: $x, y \in G$. Then

$$\lambda^G_T[^{\alpha^+}_{\alpha^-}](x) = \alpha^+ = \lambda^G_T[^{\alpha^+}_{\alpha^-}](y),$$
$$\lambda^G_I[^{\beta^-}_{\beta^+}](x) = \beta^- = \lambda^G_I[^{\beta^-}_{\beta^+}](y),$$
$$\lambda^G_F[^{\gamma^+}_{\gamma^-}](x) = \gamma^+ = \lambda^G_F[^{\gamma^+}_{\gamma^-}](y).$$

Thus

$$\min\{\lambda^G_T[^{\alpha^+}_{\alpha^-}](x), \lambda^G_T[^{\alpha^+}_{\alpha^-}](y)\} = \alpha^+,$$
$$\max\{\lambda^G_I[^{\beta^-}_{\beta^+}](x), \lambda^G_I[^{\beta^-}_{\beta^+}](y)\} = \beta^-,$$
$$\min\{\lambda^G_F[^{\gamma^+}_{\gamma^-}](x), \lambda^G_F[^{\gamma^+}_{\gamma^-}](y)\} = \gamma^+.$$

Since G is a UP-subalgebra of X, we have $x \cdot y \in G$ and so $\lambda^G_T[^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^+, \lambda^G_I[^{\beta^-}_{\beta^+}](x \cdot y) = \beta^-$, and $\lambda^G_F[^{\gamma^+}_{\gamma^-}](x \cdot y) = \gamma^+$. Hence,

$$\lambda^G_T[^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^+ \geq \alpha^+ = \min\{\lambda^G_T[^{\alpha^+}_{\alpha^-}](x), \lambda^G_T[^{\alpha^+}_{\alpha^-}](y)\},$$
$$\lambda^G_I[^{\beta^-}_{\beta^+}](x \cdot y) = \beta^- \leq \beta^- = \max\{\lambda^G_I[^{\beta^-}_{\beta^+}](x), \lambda^G_I[^{\beta^-}_{\beta^+}](y)\},$$
$$\lambda^G_F[^{\gamma^+}_{\gamma^-}](x \cdot y) = \gamma^+ \geq \gamma^+ = \min\{\lambda^G_F[^{\gamma^+}_{\gamma^-}](x), \lambda^G_F[^{\gamma^+}_{\gamma^-}](y)\}.$$

Case 2: $x \notin G$ or $y \notin G$. Then

$$\lambda_T^G\left[\begin{smallmatrix}\alpha^-\\ \alpha^-\end{smallmatrix}\right](x) = \alpha^- \text{ or } \lambda_T^G\left[\begin{smallmatrix}\alpha^+\\ \alpha^-\end{smallmatrix}\right](y) = \alpha^-,$$
$$\lambda_I^G\left[\begin{smallmatrix}\beta^-\\ \beta^+\end{smallmatrix}\right](x) = \beta^+ \text{ or } \lambda_I^G\left[\begin{smallmatrix}\beta^-\\ \beta^+\end{smallmatrix}\right](y) = \beta^+,$$
$$\lambda_F^G\left[\begin{smallmatrix}\gamma^+\\ \gamma^-\end{smallmatrix}\right](x) = \gamma^- \text{ or } \lambda_F^G\left[\begin{smallmatrix}\gamma^+\\ \gamma^-\end{smallmatrix}\right](y) = \gamma^-.$$

Thus

$$\min\{\lambda_T^G\left[\begin{smallmatrix}\alpha^+\\ \alpha^-\end{smallmatrix}\right](x), \lambda_T^G\left[\begin{smallmatrix}\alpha^+\\ \alpha^-\end{smallmatrix}\right](y)\} = \alpha^-,$$
$$\max\{\lambda_I^G\left[\begin{smallmatrix}\beta^-\\ \beta^+\end{smallmatrix}\right](x), \lambda_I^G\left[\begin{smallmatrix}\beta^-\\ \beta^+\end{smallmatrix}\right](y)\} = \beta^+,$$
$$\min\{\lambda_F^G\left[\begin{smallmatrix}\gamma^+\\ \gamma^-\end{smallmatrix}\right](x), \lambda_F^G\left[\begin{smallmatrix}\gamma^+\\ \gamma^-\end{smallmatrix}\right](y)\} = \gamma^-.$$

Therefore,

$$\lambda_T^G\left[\begin{smallmatrix}\alpha^+\\ \alpha^-\end{smallmatrix}\right](x \cdot y) \geq \alpha^- = \min\{\lambda_T^G\left[\begin{smallmatrix}\alpha^+\\ \alpha^-\end{smallmatrix}\right](x), \lambda_T^G\left[\begin{smallmatrix}\alpha^+\\ \alpha^-\end{smallmatrix}\right](y)\},$$
$$\lambda_I^G\left[\begin{smallmatrix}\beta^-\\ \beta^+\end{smallmatrix}\right](x \cdot y) \leq \beta^+ = \max\{\lambda_I^G\left[\begin{smallmatrix}\beta^-\\ \beta^+\end{smallmatrix}\right](x), \lambda_I^G\left[\begin{smallmatrix}\beta^-\\ \beta^+\end{smallmatrix}\right](y)\},$$
$$\lambda_F^G\left[\begin{smallmatrix}\gamma^+\\ \gamma^-\end{smallmatrix}\right](x \cdot y) \geq \gamma^- = \min\{\lambda_F^G\left[\begin{smallmatrix}\gamma^+\\ \gamma^-\end{smallmatrix}\right](x), \lambda_F^G\left[\begin{smallmatrix}\gamma^+\\ \gamma^-\end{smallmatrix}\right](y)\}.$$

Hence, $\Lambda^G\left[\begin{smallmatrix}\alpha^+,\beta^-,\gamma^+\\ \alpha^-,\beta^+,\gamma^-\end{smallmatrix}\right]$ is a neutrosophic UP-subalgebra of X. □

Theorem 4.1.32 *A NS $\Lambda^G\left[\begin{smallmatrix}\alpha^+,\beta^-,\gamma^+\\ \alpha^-,\beta^+,\gamma^-\end{smallmatrix}\right]$ in X is a neutrosophic near UP-filter of X if and only if a nonempty subset G of X is a near UP-filter of X.*

Proof. Assume that $\Lambda^G\left[\begin{smallmatrix}\alpha^+,\beta^-,\gamma^+\\ \alpha^-,\beta^+,\gamma^-\end{smallmatrix}\right]$ is neutrosophic near UP-filter of X. Since $\Lambda^G\left[\begin{smallmatrix}\alpha^+,\beta^-,\gamma^+\\ \alpha^-,\beta^+,\gamma^-\end{smallmatrix}\right]$ satisfies the condition (4.1.4), it follows from Lemma 4.1.30 that $0 \in G$. Next, let $x \in X$ and $y \in G$. Then $\lambda_T^G\left[\begin{smallmatrix}\alpha^+\\ \alpha^-\end{smallmatrix}\right](y) = \alpha^+$. Thus

$$\lambda_T^G\left[\begin{smallmatrix}\alpha^+\\ \alpha^-\end{smallmatrix}\right](x \cdot y) \geq \lambda_T^G\left[\begin{smallmatrix}\alpha^+\\ \alpha^-\end{smallmatrix}\right](y) = \alpha^+ \geq \lambda_T^G\left[\begin{smallmatrix}\alpha^+\\ \alpha^-\end{smallmatrix}\right](x \cdot y) \qquad ((4.1.7))$$

and so $\lambda_T^G\left[\begin{smallmatrix}\alpha^+\\ \alpha^-\end{smallmatrix}\right](x \cdot y) = \alpha^+$. Thus $x \cdot y \in G$. Hence, G is a near UP-filter of X.

Conversely, assume that G is a near UP-filter of X. Since $0 \in G$, it follows from Lemma 4.1.29 that $\Lambda^G\left[\begin{smallmatrix}\alpha^+,\beta^-,\gamma^+\\ \alpha^-,\beta^+,\gamma^-\end{smallmatrix}\right]$ satisfies the conditions (4.1.4), (4.1.5), and (4.1.6). Next, let $x, y \in X$.

Case 1: $y \in G$. Then $\lambda_T^G\left[\begin{smallmatrix}\alpha^+\\ \alpha^-\end{smallmatrix}\right](y) = \alpha^+, \lambda_I^G\left[\begin{smallmatrix}\beta^-\\ \beta^+\end{smallmatrix}\right](y) = \beta^-$, and $\lambda_F^G\left[\begin{smallmatrix}\gamma^+\\ \gamma^-\end{smallmatrix}\right](y) = \gamma^+$. Since G is a near UP-filter of X, we have $x \cdot y \in G$ and so $\lambda_T^G\left[\begin{smallmatrix}\alpha^+\\ \alpha^-\end{smallmatrix}\right](x \cdot y) = \alpha^+, \lambda_I^G\left[\begin{smallmatrix}\beta^-\\ \beta^+\end{smallmatrix}\right](x \cdot y) = \beta^-$, and $\lambda_F^G\left[\begin{smallmatrix}\gamma^+\\ \gamma^-\end{smallmatrix}\right](x \cdot y) = \gamma^+$. Thus

$$\begin{aligned}
\lambda_T^G\left[\begin{smallmatrix}\alpha^+\\ \alpha^-\end{smallmatrix}\right](x \cdot y) &= \alpha^+ \geq \alpha^+ = \lambda_T^G\left[\begin{smallmatrix}\alpha^+\\ \alpha^-\end{smallmatrix}\right](y),\\
\lambda_I^G\left[\begin{smallmatrix}\beta^-\\ \beta^+\end{smallmatrix}\right](x \cdot y) &= \beta^- \leq \beta^- = \lambda_I^G\left[\begin{smallmatrix}\beta^-\\ \beta^+\end{smallmatrix}\right](y),\\
\lambda_F^G\left[\begin{smallmatrix}\gamma^+\\ \gamma^-\end{smallmatrix}\right](x \cdot y) &= \gamma^+ \geq \gamma^+ = \lambda_F^G\left[\begin{smallmatrix}\gamma^+\\ \gamma^-\end{smallmatrix}\right](y).
\end{aligned}$$

Case 2: $y \notin G$. Then $\lambda_T^G\left[\begin{smallmatrix}\alpha^+\\ \alpha^-\end{smallmatrix}\right](y) = \alpha^-, \lambda_I^G\left[\begin{smallmatrix}\beta^-\\ \beta^+\end{smallmatrix}\right](y) = \beta^+$, and $\lambda_F^G\left[\begin{smallmatrix}\gamma^+\\ \gamma^-\end{smallmatrix}\right](y) = \gamma^-$. Thus

$$\begin{aligned}
\lambda_T^G\left[\begin{smallmatrix}\alpha^+\\ \alpha^-\end{smallmatrix}\right](x \cdot y) &\geq \alpha^- = \lambda_T^G\left[\begin{smallmatrix}\alpha^+\\ \alpha^-\end{smallmatrix}\right](y),\\
\lambda_I^G\left[\begin{smallmatrix}\beta^-\\ \beta^+\end{smallmatrix}\right](x \cdot y) &\leq \beta^+ = \lambda_I^G\left[\begin{smallmatrix}\beta^-\\ \beta^+\end{smallmatrix}\right](y),\\
\lambda_F^G\left[\begin{smallmatrix}\gamma^+\\ \gamma^-\end{smallmatrix}\right](x \cdot y) &\geq \gamma^- = \lambda_F^G\left[\begin{smallmatrix}\gamma^+\\ \gamma^-\end{smallmatrix}\right](y).
\end{aligned}$$

Hence, $\Lambda^G\left[\begin{smallmatrix}\alpha^+,\beta^-,\gamma^+\\ \alpha^-,\beta^+,\gamma^-\end{smallmatrix}\right]$ is a neutrosophic near UP-filter of X. □

Theorem 4.1.33 *A NS* $\Lambda^G\left[\begin{smallmatrix}\alpha^+,\beta^-,\gamma^+\\ \alpha^-,\beta^+,\gamma^-\end{smallmatrix}\right]$ *in* X *is a neutrosophic UP-filter of* X *if and only if a nonempty subset* G *of* X *is a UP-filter of* X.

Proof. Assume that $\Lambda^G\left[\begin{smallmatrix}\alpha^+,\beta^-,\gamma^+\\ \alpha^-,\beta^+,\gamma^-\end{smallmatrix}\right]$ is a neutrosophic UP-filter of X. Since $\Lambda^G\left[\begin{smallmatrix}\alpha^+,\beta^-,\gamma^+\\ \alpha^-,\beta^+,\gamma^-\end{smallmatrix}\right]$ satisfies the condition (4.1.4), it follows from Lemma 4.1.30 that $0 \in G$. Next, let $x, y \in X$ be such that $x \cdot y \in G$ and $x \in G$. Then $\lambda_T^G\left[\begin{smallmatrix}\alpha^+\\ \alpha^-\end{smallmatrix}\right](x \cdot y) = \alpha^+ = \lambda_T^G\left[\begin{smallmatrix}\alpha^+\\ \alpha^-\end{smallmatrix}\right](x)$. Thus

$$\lambda_T^G\left[\begin{smallmatrix}\alpha^+\\ \alpha^-\end{smallmatrix}\right](y) \geq \min\{\lambda_T^G\left[\begin{smallmatrix}\alpha^+\\ \alpha^-\end{smallmatrix}\right](x \cdot y), \lambda_T^G\left[\begin{smallmatrix}\alpha^+\\ \alpha^-\end{smallmatrix}\right](x)\} = \alpha^+ \geq \lambda_T^G\left[\begin{smallmatrix}\alpha^+\\ \alpha^-\end{smallmatrix}\right](y) \qquad ((4.1.10))$$

and so $\lambda_T^G[^{\alpha^+}_{\alpha^-}](y) = \alpha^+$. Thus $y \in G$. Hence, G is a UP-filter of X.

Conversely, assume that G is a UP-filter of X. Since $0 \in G$, it follows from Lemma 4.1.29 that $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ satisfies the conditions (4.1.4), (4.1.5), and (4.1.6). Next, let $x, y \in X$.

Case 1: $x \cdot y \in G$ and $x \in G$. Then

$$\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^+ = \lambda_T^G[^{\alpha^+}_{\alpha^-}](x),$$
$$\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot y) = \beta^- = \lambda_I^G[^{\beta^-}_{\beta^+}](x),$$
$$\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot y) = \gamma^+ = \lambda_F^G[^{\gamma^+}_{\gamma^-}](x).$$

Since G is a UP-filter of X, we have $y \in G$ and so $\lambda_T^G[^{\alpha^+}_{\alpha^-}](y) = \alpha^+$, $\lambda_I^G[^{\beta^-}_{\beta^+}](y) = \beta^-$, and $\lambda_F^G[^{\gamma^+}_{\gamma^-}](y) = \gamma^+$. Thus

$$\lambda_T^G[^{\alpha^+}_{\alpha^-}](y) = \alpha^+ \geq \alpha^+ = \min\{\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot y), \lambda_T^G[^{\alpha^+}_{\alpha^-}](x)\},$$
$$\lambda_I^G[^{\beta^-}_{\beta^+}](y) = \beta^- \leq \beta^- = \max\{\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot y), \lambda_I^G[^{\beta^-}_{\beta^+}](x)\},$$
$$\lambda_F^G[^{\gamma^+}_{\gamma^-}](y) = \gamma^+ \geq \gamma^+ = \min\{\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot y), \lambda_F^G[^{\gamma^+}_{\gamma^-}](x)\}.$$

Case 2: $x \cdot y \notin G$ or $x \notin G$. Then

$$\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^- \text{ or } \lambda_T^G[^{\alpha^+}_{\alpha^-}](x) = \alpha^-,$$
$$\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot y) = \beta^+ \text{ or } \lambda_I^G[^{\beta^-}_{\beta^+}](x) = \beta^+,$$
$$\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot y) = \gamma^- \text{ or } \lambda_F^G[^{\gamma^+}_{\gamma^-}](x) = \gamma^-.$$

Thus

$$\min\{\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot y), \lambda_T^G[^{\alpha^+}_{\alpha^-}](x)\} = \alpha^-,$$
$$\max\{\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot y), \lambda_I^G[^{\beta^-}_{\beta^+}](x)\} = \beta^+,$$

$$\min\{\lambda_F^G[^{\gamma^+}_{\gamma^-}](x\cdot y), \lambda_F^G[^{\gamma^+}_{\gamma^-}](x)\} = \gamma^-.$$

Therefore,

$$\lambda_T^G[^{\alpha^+}_{\alpha^-}](y) \geq \alpha^- = \min\{\lambda_T^G[^{\alpha^+}_{\alpha^-}](x\cdot y), \lambda_T^G[^{\alpha^+}_{\alpha^-}](x)\},$$
$$\lambda_I^G[^{\beta^-}_{\beta^+}](y) \leq \beta^+ = \max\{\lambda_I^G[^{\beta^-}_{\beta^+}](x\cdot y), \lambda_I^G[^{\beta^-}_{\beta^+}](x)\},$$
$$\lambda_F^G[^{\gamma^+}_{\gamma^-}](y) \geq \gamma^- = \max\{\lambda_F^G[^{\gamma^+}_{\gamma^-}](x\cdot y), \lambda_F^G[^{\gamma^+}_{\gamma^-}](x)\}.$$

Hence, $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a neutrosophic UP-filter of X. □

Theorem 4.1.34 *A NS $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ in X is a neutrosophic UP-ideal of X if and only if a nonempty subset G of X is a UP-ideal of X.*

Proof. Assume that $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a neutrosophic UP-ideal of X. Since $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ satisfies the condition (4.1.4), it follows from Lemma 4.1.30 that $0 \in G$. Next, let $x, y, z \in X$ be such that $x\cdot(y\cdot z) \in G$ and $y \in G$. Then $\lambda_T^G[^{\alpha^+}_{\alpha^-}](x\cdot(y\cdot z)) = \alpha^+ = \lambda_T^G[^{\alpha^+}_{\alpha^-}](y)$. Thus

$$\lambda_T^G[^{\alpha^+}_{\alpha^-}](x\cdot z) \geq \min\{\lambda_T^G[^{\alpha^+}_{\alpha^-}](x\cdot(y\cdot z)), \lambda_T^G[^{\alpha^+}_{\alpha^-}](y)\} = \alpha^+ \geq \lambda_T^G[^{\alpha^+}_{\alpha^-}](x\cdot z) \tag{(4.1.16)}$$

and so $\lambda_T^G[^{\alpha^+}_{\alpha^-}](x\cdot z) = \alpha^+$. Thus $x\cdot z \in G$. Hence, G is a UP-ideal of X.

Conversely, assume that G is a UP-ideal of X. Since $0 \in G$, it follows from Lemma 4.1.29 that $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ satisfies the conditions (4.1.4), (4.1.5), and (4.1.6). Next, let $x, y, z \in X$.

Case 1: $x\cdot(y\cdot z) \in G$ and $y \in G$. Then

$$\lambda_T^G[^{\alpha^+}_{\alpha^-}](x\cdot(y\cdot z)) = \alpha^+ = \lambda_T^G[^{\alpha^+}_{\alpha^-}](y),$$
$$\lambda_I^G[^{\beta^-}_{\beta^+}](x\cdot(y\cdot z)) = \beta^- = \lambda_I^G[^{\beta^-}_{\beta^+}](y),$$

$$\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot (y \cdot z)) = \gamma^+ = \lambda_F^G[^{\gamma^+}_{\gamma^-}](y).$$

Thus

$$\min\{\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot (y \cdot z)), \lambda_T^G[^{\alpha^+}_{\alpha^-}](y)\} = \alpha^+,$$
$$\max\{\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot (y \cdot z)), \lambda_I^G[^{\beta^-}_{\beta^+}](y)\} = \beta^-,$$
$$\min\{\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot (y \cdot z)), \lambda_F^G[^{\gamma^+}_{\gamma^-}](y)\} = \gamma^+.$$

Since G is a UP-ideal of X, we have $x \cdot z \in G$ and so $\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot z) = \alpha^+$, $\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot z) = \beta^-$, and $\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot z) = \gamma^+$. Thus

$$\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot z) = \alpha^+ \geq \alpha^+ = \min\{\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot (y \cdot z)), \lambda_T^G[^{\alpha^+}_{\alpha^-}](y)\},$$
$$\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot z) = \beta^- \leq \beta^- = \max\{\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot (y \cdot z)), \lambda_I^G[^{\beta^-}_{\beta^+}](y)\},$$
$$\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot z) = \gamma^+ \geq \gamma^+ = \min\{\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot (y \cdot z)), \lambda_F^G[^{\gamma^+}_{\gamma^-}](y)\}.$$

Case 2: $x \cdot (y \cdot z) \notin G$ or $y \notin G$. Then

$$\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot (y \cdot z)) = \alpha^- \text{ or } \lambda_T^G[^{\alpha^+}_{\alpha^-}](y) = \alpha^-,$$
$$\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot (y \cdot z)) = \beta^+ \text{ or } \lambda_I^G[^{\beta^-}_{\beta^+}](y) = \beta^+,$$
$$\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot (y \cdot z)) = \gamma^- \text{ or } \lambda_F^G[^{\gamma^+}_{\gamma^-}](y) = \gamma^-.$$

Thus

$$\min\{\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot (y \cdot z)), \lambda_T^G[^{\alpha^+}_{\alpha^-}](y)\} = \alpha^-,$$
$$\max\{\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot (y \cdot z)), \lambda_I^G[^{\beta^-}_{\beta^+}](y)\} = \beta^+,$$
$$\max\{\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot (y \cdot z)), \lambda_F^G[^{\gamma^+}_{\gamma^-}](y)\} = \gamma^-.$$

Therefore,

$$\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot z) \geq \alpha^- = \min\{\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot (y \cdot z)), \lambda_T^G[^{\alpha^+}_{\alpha^-}](y)\},$$
$$\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot z) \leq \beta^+ = \max\{\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot (y \cdot z)), \lambda_I^G[^{\beta^-}_{\beta^+}](y)\},$$
$$\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot z) \geq \gamma^- = \min\{\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot (y \cdot z)), \lambda_F^G[^{\gamma^+}_{\gamma^-}](y)\}.$$

Hence, $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a neutrosophic UP-ideal of X. □

Theorem 4.1.35 *A NS $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ in X is a neutrosophic strong UP-ideal of X if and only if a nonempty subset G of X is a strong UP-ideal of X.*

Proof. Assume that $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a neutrosophic strong UP-ideal of X. By Theorem 4.1.13, we have $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is constant, that is, $\lambda_T^G[^{\alpha^+}_{\alpha^-}]$ is constant. Since G is nonempty, we have $\lambda_T^G[^{\alpha^+}_{\alpha^-}](x) = \alpha^+$ for all $x \in X$. Thus $G = X$. Hence, G is a strong UP-ideal of X.

Conversely, assume that G is a strong UP-ideal of X. Then $G = X$, so

$$(\forall x \in X)\begin{pmatrix} \lambda_T^G[^{\alpha^+}_{\alpha^-}](x) = \alpha^+ \\ \lambda_I^G[^{\beta^-}_{\beta^+}](x) = \beta^- \\ \lambda_F^G[^{\gamma^+}_{\gamma^-}](x) = \gamma^+ \end{pmatrix}.$$

Thus $\lambda_T^G[^{\alpha^+}_{\alpha^-}], \lambda_I^G[^{\beta^-}_{\beta^+}]$, and $\lambda_F^G[^{\gamma^+}_{\gamma^-}]$ are constant, that is, $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is constant. By Theorem 4.1.13, we have $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a neutrosophic strong UP-ideal of X. □

Next, we discuss the relationships among neutrosophic UP-subalgebras (resp., neutrosophic near UP-filters, neutrosophic UP-filters, neutrosophic UP-ideals, neutrosophic strong UP-ideals) of UP-algebras and their level subsets.

Definition 4.1.36 [38] Let f be a fuzzy set in A. For any $t \in [0, 1]$, the sets

$$U(f; t) = \{x \in X \mid f(x) \geq t\},$$

$$L(f;t) = \{x \in X \mid f(x) \leq t\},$$
$$E(f;t) = \{x \in X \mid f(x) = t\}$$

are called an *upper t-level subset*, a *lower t-level subset*, and an *equal t-level subset* of f, respectively.

Theorem 4.1.37 *A NS Λ in X is a neutrosophic UP-subalgebra of X if and only if for all $\alpha, \beta, \gamma \in [0,1]$, the sets $U(\lambda_T;\alpha), L(\lambda_I;\beta)$, and $U(\lambda_F;\gamma)$ are either empty or UP-subalgebras of X.*

Proof. Assume that Λ is a neutrosophic UP-subalgebra of X. Let $\alpha, \beta, \gamma \in [0,1]$ be such that $U(\lambda_T;\alpha), L(\lambda_I;\beta)$, and $U(\lambda_F;\gamma)$ are nonempty.

Let $x, y \in U(\lambda_T;\alpha)$. Then $\lambda_T(x) \geq \alpha$ and $\lambda_T(y) \geq \alpha$, so α is an lower bound of $\{\lambda_T(x), \lambda_T(y)\}$. By (4.1.1), we have $\lambda_T(x \cdot y) \geq \min\{\lambda_T(x), \lambda_T(y)\} \geq \alpha$. Thus $x \cdot y \in U(\lambda_T;\alpha)$.

Let $x, y \in L(\lambda_I;\beta)$. Then $\lambda_I(x) \leq \beta$ and $\lambda_I(y) \leq \beta$, so β is a upper bound of $\{\lambda_I(x), \lambda_I(y)\}$. By (4.1.2), we have $\lambda_I(x \cdot y) \leq \max\{\lambda_I(x), \lambda_I(y)\} \leq \beta$. Thus $x \cdot y \in L(\lambda_I;\beta)$.

Let $x, y \in U(\lambda_F;\gamma)$. Then $\lambda_F(x) \geq \gamma$ and $\lambda_F(y) \geq \gamma$, so γ is an lower bound of $\{\lambda_F(x), \lambda_F(y)\}$. By (4.1.3), we have $\lambda_F(x \cdot y) \geq \min\{\lambda_F(x), \lambda_F(y)\} \geq \gamma$. Thus $x \cdot y \in U(\lambda_F;\gamma)$.

Hence, $U(\lambda_T;\alpha), L(\lambda_I;\beta)$, and $U(\lambda_F;\gamma)$ are UP-subalgebras of X.

Conversely, assume that for all $\alpha, \beta, \gamma \in [0,1]$, the sets $U(\lambda_T;\alpha), L(\lambda_I;\beta)$, and $U(\lambda_F;\gamma)$ are UP-subalgebras of X if $U(\lambda_T;\alpha), L(\lambda_I;\beta)$, and $U(\lambda_F;\gamma)$ are nonempty.

Let $x, y \in X$. Then $\lambda_T(x), \lambda_T(y) \in [0,1]$. Choose $\alpha = \min\{\lambda_T(x), \lambda_T(y)\}$.

Thus $\lambda_T(x) \geq \alpha$ and $\lambda_T(y) \geq \alpha$, so $x, y \in U(\lambda_T; \alpha) \neq \emptyset$. By assumption, we have $U(\lambda_T; \alpha)$ is a UP-subalgebra of X and so $x \cdot y \in U(\lambda_T; \alpha)$. Thus $\lambda_T(x \cdot y) \geq \alpha = \min\{\lambda_T(x), \lambda_T(y)\}$.

Let $x, y \in X$. Then $\lambda_I(x), \lambda_I(y) \in [0, 1]$. Choose $\beta = \max\{\lambda_I(x), \lambda_I(y)\}$. Thus $\lambda_I(x) \leq \beta$ and $\lambda_I(y) \leq \beta$, so $x, y \in L(\lambda_I; \beta) \neq \emptyset$. By assumption, we have $L(\lambda_I; \beta)$ is a UP-subalgebra of X and so $x \cdot y \in L(\lambda_I; \beta)$. Thus $\lambda_I(x \cdot y) \leq \beta = \max\{\lambda_I(x), \lambda_I(y)\}$.

Let $x, y \in X$. Then $\lambda_F(x), \lambda_F(y) \in [0, 1]$. Choose $\gamma = \min\{\lambda_F(x), \lambda_F(y)\}$. Thus $\lambda_F(x) \geq \gamma$ and $\lambda_F(y) \geq \gamma$, so $x, y \in U(\lambda_F; \gamma) \neq \emptyset$. By assumption, we have $U(\lambda_F; \gamma)$ is a UP-subalgebra of X and so $x \cdot y \in U(\lambda_F; \gamma)$. Thus $\lambda_F(x \cdot y) \geq \gamma = \min\{\lambda_F(x), \lambda_F(y)\}$.

Therefore, Λ is a neutrosophic UP-subalgebra of X. □

Theorem 4.1.38 *A NS Λ in X is a neutrosophic near UP-filter of X if and only if for all $\alpha, \beta, \gamma \in [0, 1]$, the sets $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are either empty or near UP-filters of X.*

Proof. Assume that Λ is a neutrosophic near UP-filter of X. Let $\alpha, \beta, \gamma \in [0, 1]$ be such that $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are nonempty.

Let $x \in U(\lambda_T; \alpha)$. Then $\lambda_T(x) \geq \alpha$. By (4.1.4), we have $\lambda_T(0) \geq \lambda_T(x) \geq \alpha$. Thus $0 \in U(\lambda_T; \alpha)$. Next, let $x \in X$ and $y \in U(\lambda_T; \alpha)$. Then $\lambda_T(y) \geq \alpha$. By (4.1.7), we have $\lambda_T(x \cdot y) \geq \lambda_T(y) \geq \alpha$. Thus $x \cdot y \in U(\lambda_T; \alpha)$.

Let $x \in L(\lambda_I; \beta)$. Then $\lambda_I(x) \leq \beta$. By (4.1.5), we have $\lambda_I(0) \leq \lambda_I(x) \leq \beta$. Thus $0 \in L(\lambda_I; \beta)$. Next, let $x \in X$ and $y \in L(\lambda_I; \beta)$. Then $\lambda_I(y) \leq \beta$. By (4.1.8), we have $\lambda_I(x \cdot y) \leq \lambda_I(y) \leq \beta$. Thus $x \cdot y \in L(\lambda_I; \beta)$.

Let $x \in U(\lambda_F; \gamma)$. Then $\lambda_F(x) \geq \gamma$. By (4.1.6), we have $\lambda_F(0) \geq \lambda_F(x) \geq \gamma$. Thus $0 \in U(\lambda_F; \gamma)$. Next, let $x \in X$ and $y \in U(\lambda_F; \gamma)$. Then $\lambda_F(y) \geq \gamma$. By

(4.1.9), we have $\lambda_F(x \cdot y) \geq \lambda_F(y) \geq \gamma$. Thus $x \cdot y \in U(\lambda_F; \gamma)$.

Hence, $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are near UP-filters of X.

Conversely, assume that for all $\alpha, \beta, \gamma \in [0, 1]$, the sets $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are near UP-filters of X if $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are nonempty.

Let $x \in X$. Then $\lambda_T(x) \in [0, 1]$. Choose $\alpha = \lambda_T(x)$. Thus $\lambda_T(x) \geq \alpha$, so $x \in U(\lambda_T; \alpha) \neq \emptyset$. By assumption, we have $U(\lambda_T; \alpha)$ is a near UP-filter of X and so $0 \in U(\lambda_T; \alpha)$. Thus $\lambda_T(0) \geq \alpha = \lambda_T(x)$. Next, let $x, y \in X$. Then $\lambda_T(y) \in [0, 1]$. Choose $\alpha = \lambda_T(y)$. Thus $\lambda_T(y) \geq \alpha$, so $y \in U(\lambda_T; \alpha) \neq \emptyset$. By assumption, we have $U(\lambda_T; \alpha)$ is a near UP-filter of X and so $x \cdot y \in U(\lambda_T; \alpha)$. Thus $\lambda_T(x \cdot y) \geq \alpha = \lambda_T(y)$.

Let $x \in X$. Then $\lambda_I(x) \in [0, 1]$. Choose $\beta = \lambda_I(x)$. Thus $\lambda_I(x) \leq \beta$, so $x \in L(\lambda_I; \beta) \neq \emptyset$. By assumption, we have $L(\lambda_I; \beta)$ is a near UP-filter of X and so $0 \in L(\lambda_I; \beta)$. Thus $\lambda_I(0) \leq \beta = \lambda_I(x)$. Next, let $x, y \in X$. Then $\lambda_I(y) \in [0, 1]$. Choose $\beta = \lambda_I(y)$. Thus $\lambda_I(y) \leq \beta$, so $y \in L(\lambda_I; \beta) \neq \emptyset$. By assumption, we have $L(\lambda_I; \beta)$ is a near UP-filter of X and so $x \cdot y \in L(\lambda_I; \beta)$. Thus $\lambda_I(x \cdot y) \leq \beta = \lambda_I(y)$.

Let $x \in X$. Then $\lambda_F(x) \in [0, 1]$. Choose $\gamma = \lambda_F(x)$. Thus $\lambda_F(x) \geq \gamma$, so $x \in U(\lambda_F; \gamma) \neq \emptyset$. By assumption, we have $U(\lambda_F; \gamma)$ is a near UP-filter of X and so $0 \in U(\lambda_F; \gamma)$. Thus $\lambda_F(0) \geq \gamma = \lambda_F(x)$. Next, let $x, y \in X$. Then $\lambda_F(y) \in [0, 1]$. Choose $\gamma = \lambda_F(y)$. Thus $\lambda_F(y) \geq \gamma$, so $y \in U(\lambda_F; \gamma) \neq \emptyset$. By assumption, we have $L(\lambda_F; \gamma)$ is a near UP-filter of X and so $x \cdot y \in U(\lambda_F; \gamma)$. Thus $\lambda_F(x \cdot y) \geq \gamma = \lambda_F(y)$.

Therefore, Λ is a neutrosophic near UP-filter of X. $\square$

Theorem 4.1.39 *A NS Λ in X is a neutrosophic UP-filter of X if and only if* *for all $\alpha, \beta, \gamma \in [0,1]$, the sets $U(\lambda_T;\alpha), L(\lambda_I;\beta)$, and $U(\lambda_F;\gamma)$ are either empty or UP-filters of X.*

Proof. Assume that Λ is a neutrosophic UP-filter of X. Let $\alpha, \beta, \gamma \in [0,1]$ be such that $U(\lambda_T;\alpha), L(\lambda_I;\beta)$, and $U(\lambda_F;\gamma)$ are nonempty.

Let $x \in U(\lambda_T;\alpha)$. Then $\lambda_T(x) \geq \alpha$. By (4.1.4), we have $\lambda_T(0) \geq \lambda_T(x) \geq \alpha$. Thus $0 \in U(\lambda_T;\alpha)$. Next, let $x, y \in X$ be such that $x \cdot y \in U(\lambda_T;\alpha)$ and $x \in U(\lambda_T;\alpha)$. Then $\lambda_T(x \cdot y) \geq \alpha$ and $\lambda_T(x) \geq \alpha$, so α is an lower bound of $\{\lambda_T(x \cdot y), \lambda_T(x)\}$. By (4.1.10), we have $\lambda_T(y) \geq \min\{\lambda_T(x \cdot y), \lambda_T(x)\} \geq \alpha$. Thus $y \in U(\lambda_T;\alpha)$.

Let $x \in L(\lambda_I;\beta)$. Then $\lambda_I(x) \leq \beta$. By (4.1.5), we have $\lambda_I(0) \leq \lambda_I(x) \leq \beta$. Thus $0 \in L(\lambda_I;\beta)$. Next, let $x, y \in X$ be such that $x \cdot y \in L(\lambda_I;\beta)$ and $x \in L(\lambda_I;\beta)$. Then $\lambda_I(x \cdot y) \leq \beta$ and $\lambda_I(x) \leq \beta$, so β is a upper bound of $\{\lambda_I(x \cdot y), \lambda_I(x)\}$. By (4.1.11), we have $\lambda_I(y) \leq \max\{\lambda_I(x \cdot y), \lambda_I(x)\} \leq \beta$ Thus $y \in L(\lambda_I;\beta)$.

Let $x \in U(\lambda_F;\gamma)$. Then $\lambda_F(x) \geq \gamma$. By (4.1.6), we have $\lambda_F(0) \geq \lambda_F(x) \geq \gamma$. Thus $0 \in U(\lambda_F;\gamma)$. Next, let $x, y \in X$ be such that $x \cdot y \in U(\lambda_F;\gamma)$ and $x \in U(\lambda_F;\gamma)$. Then $\lambda_F(x \cdot y) \geq \gamma$ and $\lambda_F(x) \geq \gamma$, so γ is an lower bound of $\{\lambda_F(x \cdot y), \lambda_F(x)\}$. By (4.1.12), we have $\lambda_F(y) \geq \min\{\lambda_F(x \cdot y), \lambda_F(x)\} \geq \gamma$. Thus $y \in U(\lambda_F;\gamma)$.

Hence, $U(\lambda_T;\alpha), L(\lambda_I;\beta)$, and $U(\lambda_F;\gamma)$ are UP-filters of X.

Conversely, assume that for all $\alpha, \beta, \gamma \in [0,1]$, the sets $U(\lambda_T;\alpha), L(\lambda_I;\beta)$, and $U(\lambda_F;\gamma)$ are UP-filters of X if $U(\lambda_T;\alpha), L(\lambda_I;\beta)$, and $U(\lambda_F;\gamma)$ are nonempty.

Let $x \in X$. Then $\lambda_T(x) \in [0,1]$. Choose $\alpha = \lambda_T(x)$. Thus $\lambda_T(x) \geq \alpha$, so $x \in U(\lambda_T;\alpha) \neq \emptyset$. By assumption, we have $U(\lambda_T;\alpha)$ is a UP-filter of X and so $0 \in U(\lambda_T;\alpha)$. Thus $\lambda_T(0) \geq \alpha = \lambda_T(x)$. Next, let $x, y \in X$. Then

$\lambda_T(x \cdot y), \lambda_T(x) \in [0,1]$. Choose $\alpha = \min\{\lambda_T(x \cdot y), \lambda_T(x)\}$. Thus $\lambda_T(x \cdot y) \geq \alpha$ and $\lambda_T(x) \geq \alpha$, so $x \cdot y, x \in U(\lambda_T; \alpha) \neq \emptyset$. By assumption, we have $U(\lambda_T; \alpha)$ is a UP-filter of X and so $y \in U(\lambda_T; \alpha)$. Thus $\lambda_T(y) \geq \alpha = \min\{\lambda_T(x \cdot y), \lambda_T(x)\}$.

Let $x \in X$. Then $\lambda_I(x) \in [0,1]$. Choose $\beta = \lambda_I(x)$. Thus $\lambda_I(x) \leq \beta$, so $x \in L(\lambda_I; \beta) \neq \emptyset$. By assumption, we have $L(\lambda_I; \beta)$ is a UP-filter of X and so $0 \in L(\lambda_I; \beta)$. Thus $\lambda_I(0) \leq \beta = \lambda_I(x)$. Next, let $x, y \in X$. Then $\lambda_I(x \cdot y), \lambda_I(x) \in [0,1]$. Choose $\beta = \max\{\lambda_I(x \cdot y), \lambda_I(x)\}$. Thus $\lambda_I(x \cdot y) \leq \beta$ and $\lambda_I(x) \leq \beta$, so $x \cdot y, x \in L(\lambda_I; \beta) \neq \emptyset$. By assumption, we have $L(\lambda_I; \beta)$ is a UP-filter of X and so $y \in L(\lambda_I; \beta)$. Thus $\lambda_I(y) \leq \beta = \max\{\lambda_I(x \cdot y), \lambda_I(x)\}$.

Let $x \in X$. Then $\lambda_F(x) \in [0,1]$. Choose $\gamma = \lambda_F(x)$. Thus $\lambda_F(x) \geq \gamma$, so $x \in U(\lambda_F; \gamma) \neq \emptyset$. By assumption, we have $U(\lambda_F; \gamma)$ is a UP-filter of X and so $0 \in U(\lambda_F; \gamma)$. Thus $\lambda_F(0) \geq \gamma = \lambda_F(x)$. Next, let $x, y \in X$. Then $\lambda_F(x \cdot y), \lambda_F(x) \in [0,1]$. Choose $\gamma = \min\{\lambda_F(x \cdot y), \lambda_F(x)\}$. Thus $\lambda_F(x \cdot y) \geq \gamma$ and $\lambda_F(x) \geq \gamma$, so $x \cdot y, x \in U(\lambda_F; \gamma) \neq \emptyset$. By assumption, we have $U(\lambda_F; \gamma)$ is a UP-filter of X and so $y \in U(\lambda_F; \gamma)$. Thus $\lambda_F(y) \geq \gamma = \min\{\lambda_F(x \cdot y), \lambda_F(x)\}$.

Therefore, Λ is a neutrosophic UP-filter of X. □

Theorem 4.1.40 *A NS Λ in X is a neutrosophic UP-ideal of X if and only if for all $\alpha, \beta, \gamma \in [0,1]$, the sets $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are either empty or UP-ideals of X.*

Proof. Assume that Λ is a neutrosophic UP-ideal of X. Let $\alpha, \beta, \gamma \in [0,1]$ be such that $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are nonempty.

Let $x \in U(\lambda_T; \alpha)$. Then $\lambda_T(x) \geq \alpha$. By (4.1.4), we have $\lambda_T(0) \geq \lambda_T(x) \geq \alpha$. Thus $0 \in U(\lambda_T; \alpha)$. Next, let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in U(\lambda_T; \alpha)$ and $y \in U(\lambda_T; \alpha)$. Then $\lambda_T(x \cdot (y \cdot z)) \geq \alpha$ and $\lambda_T(y) \geq \alpha$, so α is an lower bound of $\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\}$. By (4.1.13), we have $\lambda_T(x \cdot z) \geq \min\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\} \geq \alpha$. Thus $x \cdot z \in U(\lambda_T; \alpha)$.

Let $x \in L(\lambda_I; \alpha)$. Then $\lambda_I(x) \leq \beta$. By (4.1.5), we have $\lambda_I(0) \leq \lambda_I(x) \leq \beta$. Thus $0 \in L(\lambda_I; \beta)$. Next, let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in L(\lambda_I; \beta)$ and $y \in L(\lambda_I; \beta)$. Then $\lambda_I(x \cdot (y \cdot z)) \leq \beta$ and $\lambda_I(y) \leq \beta$, so β is a upper bound of $\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\}$. By (4.1.14), we have $\lambda_I(x \cdot z) \leq \max\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\} \leq \beta$. Thus $x \cdot z \in L(\lambda_I; \beta)$.

Let $x \in U(\lambda_F; \gamma)$. Then $\lambda_F(x) \geq \gamma$. By (4.1.6), we have $\lambda_F(0) \geq \lambda_F(x) \geq \gamma$. Thus $0 \in U(\lambda_F; \gamma)$. Next, let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in U(\lambda_F; \gamma)$ and $y \in U(\lambda_F; \gamma)$. Then $\lambda_F(x \cdot (y \cdot z)) \geq \gamma$ and $\lambda_F(y) \geq \gamma$, so γ is an lower bound of $\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}$. By (4.1.15), we have $\lambda_F(x \cdot z) \geq \min\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\} \geq \gamma$. Thus $x \cdot z \in U(\lambda_F; \gamma)$.

Hence, $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are UP-ideals of X.

Conversely, assume that for all $\alpha, \beta, \gamma \in [0, 1]$, the sets $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are UP-ideals of X if $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are nonempty.

Let $x \in X$. Then $\lambda_T(x) \in [0, 1]$. Choose $\alpha = \lambda_T(x)$. Thus $\lambda_T(x) \geq \alpha$, so $x \in U(\lambda_T; \alpha) \neq \emptyset$. By assumption, we have $U(\lambda_T; \alpha)$ is a UP-ideal of X and so $0 \in U(\lambda_T; \alpha)$. Thus $\lambda_T(0) \geq \alpha = \lambda_T(x)$. Next, let $x, y, z \in X$. Then $\lambda_T(x \cdot (y \cdot z)), \lambda_T(y) \in [0, 1]$. Choose $\alpha = \min\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\}$. Thus $\lambda_T(x \cdot (y \cdot z)) \geq \alpha$ and $\lambda_T(y) \geq \alpha$, so $x \cdot (y \cdot z), y \in U(\lambda_T; \alpha) \neq \emptyset$. By assumption, we have $U(\lambda_T; \alpha)$ is a UP-ideal of X and so $x \cdot z \in U(\lambda_T; \alpha)$. Thus $\lambda_T(x \cdot z) \geq \alpha = \min\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\}$.

Let $x \in X$. Then $\lambda_I(x) \in [0, 1]$. Choose $\beta = \lambda_I(x)$. Thus $\lambda_I(x) \leq \beta$, so $x \in L(\lambda_I; \beta) \neq \emptyset$. By assumption, we have $L(\lambda_I; \beta)$ is a UP-ideal of X and so $0 \in L(\lambda_I; \beta)$. Thus $\lambda_I(0) \leq \beta = \lambda_I(x)$. Next, let $x, y, z \in X$. Then $\lambda_I(x \cdot (y \cdot z)), \lambda_I(y) \in [0, 1]$. Choose $\beta = \max\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\}$. Thus $\lambda_I(x \cdot (y \cdot z)) \leq \beta$ and $\lambda_I(y) \leq \beta$, so $x \cdot (y \cdot z), y \in L(\lambda_I; \beta) \neq \emptyset$. By assumption, we have $L(\lambda_I; \beta)$ is a UP-ideal of X and so $x \cdot z \in L(\lambda_I; \beta)$. Thus $\lambda_I(x \cdot z) \leq \beta =$

$\max\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\}$.

Let $x \in X$. Then $\lambda_F(x) \in [0,1]$. Choose $\gamma = \lambda_F(x)$. Thus $\lambda_F(x) \geq \gamma$, so $x \in U(\lambda_F;\gamma) \neq \emptyset$. By assumption, we have $U(\lambda_F;\gamma)$ is a UP-ideal of X and so $0 \in U(\lambda_F;\gamma)$. Thus $\lambda_F(0) \geq \gamma = \lambda_F(x)$. Next, let $x, y, z \in X$. Then $\lambda_F(x \cdot (y \cdot z)), \lambda_F(y) \in [0,1]$. Choose $\gamma = \min\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}$. Thus $\lambda_F(x \cdot (y \cdot z)) \geq \gamma$ and $\lambda_F(y) \geq \gamma$, so $x \cdot (y \cdot z), y \in U(\lambda_F;\gamma) \neq \emptyset$. By assumption, we have $U(\lambda_F;\gamma)$ is a UP-ideal of X and so $x \cdot z \in U(\lambda_F;\gamma)$. Thus $\lambda_F(x \cdot z) \geq \gamma = \min\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}$.

Therefore, Λ is a neutrosophic UP-ideal of X. □

Theorem 4.1.41 *A NS Λ in X is a neutrosophic strong UP-ideal of X if and only if the sets $E(\lambda_T;\lambda_T(0)), E(\lambda_I;\lambda_I(0))$, and $E(\lambda_F;\lambda_F(0))$ are strong UP-ideals of X.*

Proof. Assume that Λ is a neutrosophic strong UP-ideal of X. By Theorem 4.1.13, we have Λ is constant, that is, λ_T, λ_I, and λ_F are constant. Thus

$$(\forall x \in X)\begin{pmatrix}\lambda_T(x) = \lambda_T(0)\\ \lambda_I(x) = \lambda_I(0)\\ \lambda_F(x) = \lambda_F(0)\end{pmatrix}.$$

Hence, $E(\lambda_T;\lambda_T(0)) = X, E(\lambda_I;\lambda_I(0)) = X$, and $E(\lambda_F;\lambda_F(0)) = X$ and so $E(\lambda_T;\lambda_T(0))$, $E(\lambda_I;\lambda_I(0))$, and $E(\lambda_F;\lambda_F(0))$ are strong UP-ideals of X.

Conversely, assume that $E(\lambda_T;\lambda_T(0)), E(\lambda_I;\lambda_I(0))$, and $E(\lambda_F;\lambda_F(0))$ are strong UP-ideals of X. Then $E(\lambda_T;\lambda_T(0)) = X, E(\lambda_I;\lambda_I(0)) = X$, $E(\lambda_F;\lambda_F(0))$

$= X$ and so

$$(\forall x \in X)\begin{pmatrix}\lambda_T(x) = \lambda_T(0)\\ \lambda_I(x) = \lambda_I(0)\\ \lambda_F(x) = \lambda_F(0)\end{pmatrix}.$$

Thus λ_T, λ_I, and λ_F are constant, that is, Λ is constant. By Theorem 4.1.13, we have Λ is a neutrosophic strong UP-ideal of X. $\square$

Definition 4.1.42 Let Λ be a NS in X. For $\alpha, \beta, \gamma \in [0, 1]$, the sets

$$ULU_\Lambda(\alpha, \beta, \gamma) = \{x \in X \mid \lambda_T(x) \geq \alpha, \lambda_I(x) \leq \beta, \lambda_F(x) \geq \gamma\},$$
$$LUL_\Lambda(\alpha, \beta, \gamma) = \{x \in X \mid \lambda_T(x) \leq \alpha, \lambda_I(x) \geq \beta, \lambda_F(x) \leq \gamma\},$$
$$E_\Lambda(\alpha, \beta, \gamma) = \{x \in X \mid \lambda_T(x) = \alpha, \lambda_I(x) = \beta, \lambda_F(x) = \gamma\}$$

are called a *ULU-(α, β, γ)-level subset*, an *LUL-(α, β, γ)-level subset*, and an *E-(α, β, γ)-level subset* of Λ, respectively. Then we see that

$$ULU_\Lambda(\alpha, \beta, \gamma) = U(\lambda_T; \alpha) \cap L(\lambda_I; \beta) \cap U(\lambda_F; \gamma),$$
$$LUL_\Lambda(\alpha, \beta, \gamma) = L(\lambda_T; \alpha) \cap U(\lambda_I; \beta) \cap L(\lambda_F; \gamma),$$
$$E_\Lambda(\alpha, \beta, \gamma) = E(\lambda_T; \alpha) \cap E(\lambda_I; \beta) \cap E(\lambda_F; \gamma).$$

Corollary 4.1.43 *A NS Λ in X is a neutrosophic UP-subalgebra of X if and only if for all $\alpha, \beta, \gamma \in [0, 1]$, $ULU_\Lambda(\alpha, \beta, \gamma)$ is a UP-subalgebra of X where $ULU_\Lambda(\alpha, \beta, \gamma)$ is nonempty.*

Proof. It is straightforward by Theorems 3.0.6 and 4.1.37. $\square$

Corollary 4.1.44 *A NS Λ in X is a neutrosophic near UP-filter of X if and only if for all $\alpha, \beta, \gamma \in [0, 1]$, $ULU_\Lambda(\alpha, \beta, \gamma)$ is a near UP-filter of X where $ULU_\Lambda(\alpha, \beta, \gamma)$ is nonempty.*

Proof. It is straightforward by Theorems 3.0.6 and 4.1.38. □

Corollary 4.1.45 *A NS Λ in X is a neutrosophic UP-filter of X if and only if for all $\alpha, \beta, \gamma \in [0, 1]$, $ULU_\Lambda(\alpha, \beta, \gamma)$ is a UP-filter of X where $ULU_\Lambda(\alpha, \beta, \gamma)$ is nonempty.*

Proof. It is straightforward by Theorems 3.0.6 and 4.1.39. □

Corollary 4.1.46 *A NS Λ in X is a neutrosophic UP-ideal of X if and only if for all $\alpha, \beta, \gamma \in [0, 1]$, $ULU_\Lambda(\alpha, \beta, \gamma)$ is a UP-ideal of X where $ULU_\Lambda(\alpha, \beta, \gamma)$ is nonempty.*

Proof. It is straightforward by Theorems 3.0.6 and 4.1.40. □

Corollary 4.1.47 *A NS Λ in X is a neutrosophic strong UP-ideal of X if and only if $E_\Lambda(\lambda_T(0), \lambda_I(0), \lambda_F(0))$ is a strong UP-ideal of X.*

Proof. It is straightforward by Theorems 3.0.6 and 4.1.41. □

4.2 Special neutrosophic sets in UP-algebras

In this section, we introduce the parallel concepts of special neutrosophic UP-subalgebras, special neutrosophic near UP-filters, special neutrosophic UP-filters, special neutrosophic UP-ideals, and special neutrosophic strong UP-ideals of UP-algebras, provide the necessary examples, investigate their properties, and prove their generalizations.

Definition 4.2.1 A NS Λ in X is called an *special neutrosophic UP-subalgebra* of X if it satisfies the following conditions:

$$(\forall x, y \in X)(\lambda_T(x \cdot y) \leq \max\{\lambda_T(x), \lambda_T(y)\}), \tag{4.2.1}$$

$$(\forall x, y \in X)(\lambda_I(x \cdot y) \geq \min\{\lambda_I(x), \lambda_I(y)\}), \text{ and} \tag{4.2.2}$$

$$(\forall x, y \in X)(\lambda_F(x \cdot y) \leq \max\{\lambda_F(x), \lambda_F(y)\}). \tag{4.2.3}$$

Example 4.2.2 Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	1	0	4
2	0	0	0	0	4
3	0	1	1	0	4
4	0	3	3	3	0

We define a NS Λ in X as follows:

$$\lambda_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.2 & 0.3 & 0.5 & 0.7 & 0.8 \end{pmatrix}, \lambda_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 0.7 & 0.6 & 0.5 & 0.2 \end{pmatrix},$$

$$\lambda_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.1 & 0.4 & 0.6 & 0.7 & 0.9 \end{pmatrix}.$$

Hence, Λ is a special neutrosophic UP-subalgebra of X.

Definition 4.2.3 A NS Λ in X is called an *special neutrosophic near UP-filter* of X if it satisfies the following conditions:

$$(\forall x \in X)(\lambda_T(0) \leq \lambda_T(x)), \tag{4.2.4}$$

$$(\forall x \in X)(\lambda_I(0) \geq \lambda_I(x)), \tag{4.2.5}$$

$$(\forall x \in X)(\lambda_F(0) \leq \lambda_F(x)), \tag{4.2.6}$$

$$(\forall x, y \in X)(\lambda_T(x \cdot y) \leq \lambda_T(y)), \tag{4.2.7}$$

$$(\forall x, y \in X)(\lambda_I(x \cdot y) \geq \lambda_I(y)), \text{ and} \tag{4.2.8}$$

$$(\forall x, y \in X)(\lambda_F(x \cdot y) \leq \lambda_F(y)). \tag{4.2.9}$$

Example 4.2.4 Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	2	4
2	0	1	0	1	4
3	0	0	0	0	4
4	0	1	2	3	0

We define a NS Λ in X as follows:

$$\lambda_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 0.3 & 0.5 & 0.6 & 0.2 \end{pmatrix}, \lambda_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.9 & 0.8 & 0.7 & 0.3 & 0.4 \end{pmatrix},$$

$$\lambda_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.1 & 0.2 & 0.6 & 0.7 & 0.5 \end{pmatrix}.$$

Hence, Λ is a special neutrosophic near UP-filter of X.

Definition 4.2.5 A NS Λ in X is called an *special neutrosophic UP-filter* of X if it satisfies the following conditions: (4.2.4), (4.2.5), (4.2.6),

$$(\forall x, y \in X)(\lambda_T(y) \leq \max\{\lambda_T(x \cdot y), \lambda_T(x)\}), \tag{4.2.10}$$

$$(\forall x, y \in X)(\lambda_I(y) \geq \min\{\lambda_I(x \cdot y), \lambda_I(x)\}), \text{ and} \tag{4.2.11}$$

$$(\forall x, y \in X)(\lambda_F(y) \leq \max\{\lambda_F(x \cdot y), \lambda_F(x)\}). \tag{4.2.12}$$

Example 4.2.6 Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0

and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	2	4
2	0	1	0	1	4
3	0	0	0	0	4
4	0	1	1	1	0

We define a NS Λ in X as follows:

$$\lambda_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.1 & 0.5 & 0.4 & 0.5 & 0.8 \end{pmatrix}, \lambda_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.8 & 0.3 & 0.5 & 0.3 & 0.4 \end{pmatrix},$$

$$\lambda_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.2 & 0.6 & 0.4 & 0.6 & 0.3 \end{pmatrix}.$$

Hence, Λ is a special neutrosophic UP-filter of X.

Definition 4.2.7 A NS Λ in X is called an *special neutrosophic UP-ideal* of X if it satisfies the following conditions: (4.2.4), (4.2.5), (4.2.6),

$$(\forall x, y, z \in X)(\lambda_T(x \cdot z) \leq \max\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\}), \tag{4.2.13}$$

$$(\forall x, y, z \in X)(\lambda_I(x \cdot z) \geq \min\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\}), \text{ and} \tag{4.2.14}$$

$$(\forall x, y, z \in X)(\lambda_F(x \cdot z) \leq \max\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}). \tag{4.2.15}$$

Example 4.2.8 Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0

and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	0	0	0	4
3	0	0	2	0	4
4	0	0	0	0	0

We define a NS Λ in X as follows:

$$\lambda_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 0.3 & 0.5 & 0.4 & 0.6 \end{pmatrix}, \lambda_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 0.7 & 0.4 & 0.7 & 0.3 \end{pmatrix},$$

$$\lambda_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.1 & 0.2 & 0.7 & 0.3 & 0.9 \end{pmatrix}.$$

Hence, Λ is a special neutrosophic UP-ideal of X.

Definition 4.2.9 A NS Λ in X is called an *special neutrosophic strong UP-ideal* of X if it satisfies the following conditions: (4.2.4), (4.2.5), (4.2.6),

$$(\forall x, y, z \in X)(\lambda_T(x) \leq \max\{\lambda_T((z \cdot y) \cdot (z \cdot x)), \lambda_T(y)\}), \tag{4.2.16}$$

$$(\forall x, y, z \in X)(\lambda_I(x) \geq \min\{\lambda_I((z \cdot y) \cdot (z \cdot x)), \lambda_I(y)\}), \text{ and} \tag{4.2.17}$$

$$(\forall x, y, z \in X)(\lambda_F(x) \leq \max\{\lambda_F((z \cdot y) \cdot (z \cdot x)), \lambda_F(y)\}). \tag{4.2.18}$$

Example 4.2.10 Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0

and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	0
2	0	1	0	3	4
3	0	1	0	0	4
4	0	1	0	3	0

We define a NS Λ in X as follows:

$$(\forall x \in X) \begin{pmatrix} \lambda_T(x) = 0.5 \\ \lambda_I(x) = 0.4 \\ \lambda_F(x) = 0.7 \end{pmatrix}.$$

Hence, Λ is a special neutrosophic strong UP-ideal X.

Theorem 4.2.11 *Every special neutrosophic UP-subalgebra of X satisfies the conditions* (4.2.4), (4.2.5), *and* (4.2.6).

Proof. Assume that Λ is a special neutrosophic UP-subalgebra of X. Then for all $x \in X$,

$$\lambda_T(0) = \lambda_T(x \cdot x) \leq \max\{\lambda_T(x), \lambda_T(x)\} = \lambda_T(x), \qquad ((3.0.1) \text{ and } (4.2.1))$$

$$\lambda_I(0) = \lambda_I(x \cdot x) \geq \min\{\lambda_I(x), \lambda_I(x)\} = \lambda_I(x), \qquad ((3.0.1) \text{ and } (4.2.2))$$

$$\lambda_F(0) = \lambda_F(x \cdot x) \leq \max\{\lambda_F(x), \lambda_F(x)\} = \lambda_F(x). \qquad ((3.0.1) \text{ and } (4.2.3))$$

Hence, Λ satisfies the conditions (4.2.4), (4.2.5), and (4.2.6). □

By Lemma 2.0.5 (1) and (4), we have the following five theorems.

Theorem 4.2.12 *A NS Λ in X is a neutrosophic UP-subalgebra of X if and only if $\overline{\Lambda}$ is a special neutrosophic UP-subalgebra of X.*

Theorem 4.2.13 *A NS Λ in X is a neutrosophic near UP-filter of X if and only if $\overline{\Lambda}$ is a special neutrosophic near UP-filter of X.*

Theorem 4.2.14 *A NS Λ in X is a neutrosophic UP-filter of X if and only if $\overline{\Lambda}$ is a special neutrosophic UP-filter of X.*

Theorem 4.2.15 *A NS Λ in X is a neutrosophic UP-ideal of X if and only if $\overline{\Lambda}$ is a special neutrosophic UP-ideal of X.*

Theorem 4.2.16 *A NS Λ in X is a neutrosophic strong UP-ideal of X if and only if $\overline{\Lambda}$ is a special neutrosophic strong UP-ideal of X.*

Theorem 4.2.17 *A NS Λ in X is constant if and only if it is a special neutrosophic strong UP-ideal of X.*

Proof. It is straightforward by Remark 2.0.15 and Theorems 4.1.13 and 4.2.16. □

Corollary 4.2.18 *Neutrosophic strongly UP-ideals, special neutrosophic strong UP-ideals, and constant neutrosophic sets coincide.*

Proof. It is straightforward by Theorems 4.1.13 and 4.2.17. □

Theorem 4.2.19 *If Λ is a special neutrosophic UP-subalgebra of X satisfying the following condition:*

$$(\forall x, y \in X)\left(x \cdot y \neq 0 \Rightarrow \begin{cases} \lambda_T(x) \leq \lambda_T(y) \\ \lambda_I(x) \geq \lambda_I(y) \\ \lambda_F(x) \leq \lambda_F(y) \end{cases} \right), \tag{4.2.19}$$

then Λ is a special neutrosophic near UP-filter of X.

Proof. Assume that Λ is a special neutrosophic UP-subalgebra of X satisfying the condition (4.2.19). By Theorem 4.2.11, we have Λ satisfies the conditions (4.2.4), (4.2.5), and (4.2.6). Next, let $x, y \in X$.

Case 1: $x \cdot y = 0$. Then

$$\lambda_T(x \cdot y) = \lambda_T(0) \leq \lambda_T(y), \qquad ((4.2.4))$$

$$\lambda_I(x \cdot y) = \lambda_I(0) \geq \lambda_I(y), \qquad ((4.2.5))$$

$$\lambda_F(x \cdot y) = \lambda_F(0) \leq \lambda_F(y). \qquad ((4.2.6))$$

Case 2: $x \cdot y \neq 0$. Then

$$\lambda_T(x \cdot y) \leq \max\{\lambda_T(x), \lambda_T(y)\} = \lambda_T(y), \qquad ((4.2.1) \text{ and } (4.2.19) \text{ for } \lambda_T)$$

$$\lambda_I(x \cdot y) \geq \min\{\lambda_I(x), \lambda_I(y)\} = \lambda_I(y), \qquad ((4.2.2) \text{ and } (4.2.19) \text{ for } \lambda_I)$$

$$\lambda_F(x \cdot y) \leq \max\{\lambda_F(x), \lambda_F(y)\} = \lambda_F(y). \qquad ((4.2.3) \text{ and } (4.2.19) \text{ for } \lambda_F)$$

Hence, Λ is a special neutrosophic near UP-filter of X. □

Theorem 4.2.20 *If Λ is a special neutrosophic near UP-filter of X satisfying the following condition:*

$$\lambda_T = \lambda_I = \lambda_F, \qquad (4.2.20)$$

then Λ is a special neutrosophic strong UP-ideal of X.

Proof. Assume that Λ is a special neutrosophic near UP-filter of X satisfying the condition (4.2.20). Then Λ satisfies the conditions (4.2.4), (4.2.5), and (4.2.6). let $x \in X$. Then Then

$$\lambda_T(0) \leq \lambda_T(x) = \lambda_I(x) \leq \lambda_I(0) = \lambda_T(0),$$

$$\lambda_I(0) \geq \lambda_I(x) = \lambda_T(x) \geq \lambda_T(0) = \lambda_I(0),$$

$$\lambda_F(0) \leq \lambda_F(x) = \lambda_I(x) \leq \lambda_I(0) = \lambda_F(0).$$

Thus $\lambda_T(0) = \lambda_T(x), \lambda_I(0) = \lambda_I(x)$, and $\lambda_F(0) = \lambda_F(x)$, that is, Λ is constant. By theorem 4.2.17, we have Λ is a special neutrosophic strong UP-ideal of X. □

Theorem 4.2.21 *If Λ is a special neutrosophic UP-filter of X satisfying the following condition:*

$$(\forall x, y, z \in X) \begin{pmatrix} \lambda_T(y \cdot (x \cdot z)) = \lambda_T(x \cdot (y \cdot z)) \\ \lambda_I(y \cdot (x \cdot z)) = \lambda_I(x \cdot (y \cdot z)) \\ \lambda_F(y \cdot (x \cdot z)) = \lambda_F(x \cdot (y \cdot z)) \end{pmatrix}, \qquad (4.2.21)$$

then Λ is a special neutrosophic UP-ideal of X.

Proof. Assume that Λ is a special neutrosophic UP-filter of X satisfying the condition (4.2.21). Then Λ satisfies the conditions (4.2.4), (4.2.5), and (4.2.6). Next, let $x, y, z \in X$. Then

$$\begin{aligned}
\lambda_T(x \cdot z) &\leq \max\{\lambda_T(y \cdot (x \cdot z)), \lambda_T(y)\} && ((4.2.10)) \\
&= \max\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\}, && ((4.2.21) \text{ for } \lambda_T) \\
\lambda_I(x \cdot z) &\geq \min\{\lambda_I(y \cdot (x \cdot z)), \lambda_I(y)\} && ((4.2.11)) \\
&= \min\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\}, && ((4.2.21) \text{ for } \lambda_I) \\
\lambda_F(x \cdot z) &\leq \max\{\lambda_F(y \cdot (x \cdot z)), \lambda_F(y)\} && ((4.2.12)) \\
&= \max\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}. && ((4.2.21) \text{ for } \lambda_F)
\end{aligned}$$

Hence, Λ is a special neutrosophic UP-ideal of X. □

Theorem 4.2.22 *If Λ is a NS in X satisfying the following condition:*

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} \lambda_T(z) \leq \max\{\lambda_T(x), \lambda_T(y)\} \\ \lambda_I(z) \geq \min\{\lambda_I(x), \lambda_I(y)\} \\ \lambda_F(z) \leq \max\{\lambda_F(x), \lambda_F(y)\} \end{cases} \right), \tag{4.2.22}$$

then Λ is a special neutrosophic UP-subalgebra of X.

Proof. Assume that Λ is a NS in X satisfying the condition (4.2.22). Let $x, y \in X$. By (3.0.1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \geq x \cdot y$. It follows from (4.2.22) that

$$\lambda_T(x \cdot y) \leq \max\{\lambda_T(x), \lambda_T(y)\},$$
$$\lambda_I(x \cdot y) \geq \min\{\lambda_I(x), \lambda_I(y)\},$$
$$\lambda_F(x \cdot y) \leq \max\{\lambda_F(x), \lambda_F(y)\}.$$

Hence, Λ is a special neutrosophic UP-subalgebra of X. □

Theorem 4.2.23 *If Λ is a NS in X satisfying the following condition:*

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} \lambda_T(y) \leq \max\{\lambda_T(z), \lambda_T(x)\} \\ \lambda_I(y) \geq \min\{\lambda_I(z), \lambda_I(x)\} \\ \lambda_F(y) \leq \max\{\lambda_F(z), \lambda_F(x)\} \end{cases} \right), \tag{4.2.23}$$

then Λ is a special neutrosophic UP-filter of X.

Proof. Assume that Λ is a NS in X satisfying the condition (4.2.23). Let $x \in X$. By (UP-3), we have $x \cdot (x \cdot 0) = 0$, that is, $x \leq x \cdot 0$. It follows from (4.2.23) that

$$\lambda_T(0) \leq \max\{\lambda_T(x), \lambda_T(x)\} = \lambda_T(x),$$

$$\lambda_I(0) \geq \min\{\lambda_I(x), \lambda_I(x)\} = \lambda_I(x),$$

$$\lambda_F(0) \leq \max\{\lambda_F(x), \lambda_F(x)\} = \lambda_F(x).$$

Next, let $x, y \in X$. By (3.0.1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \geq x \cdot y$. It follows from (4.2.23) that

$$\lambda_T(y) \leq \max\{\lambda_T(x \cdot y), \lambda_T(x)\},$$

$$\lambda_I(y) \geq \min\{\lambda_I(x \cdot y), \lambda_I(x)\},$$

$$\lambda_F(y) \leq \max\{\lambda_F(x \cdot y), \lambda_F(x)\}.$$

Hence, Λ is a special neutrosophic UP-filter of X. □

Theorem 4.2.24 *If Λ is a NS in X satisfying the following condition:*

$$(\forall a, x, y, z \in X)\left(a \leq x \cdot (y \cdot z) \Rightarrow \begin{cases} \lambda_T(x \cdot z) \leq \max\{\lambda_T(a), \lambda_T(y)\} \\ \lambda_I(x \cdot z) \geq \min\{\lambda_I(a), \lambda_I(y)\} \\ \lambda_F(x \cdot z) \leq \max\{\lambda_F(a), \lambda_F(y)\} \end{cases}\right), \tag{4.2.24}$$

then Λ is a special neutrosophic UP-ideal of X.

Proof. Assume that Λ is a NS in X satisfying the condition (4.2.24). Let $x \in X$. By (UP-3), we have $x \cdot (0 \cdot (x \cdot 0)) = 0$, that is, $x \leq 0 \cdot (x \cdot 0)$. It follows from (4.2.24) that

$$\lambda_T(0) = \lambda_T(0 \cdot 0) \leq \max\{\lambda_T(x), \lambda_T(x)\} = \lambda_T(x), \quad \text{((UP-2))}$$

$$\lambda_I(0) = \lambda_I(0 \cdot 0) \geq \min\{\lambda_I(x), \lambda_I(x)\} = \lambda_I(x), \quad \text{((UP-2))}$$

$$\lambda_F(0) = \lambda_F(0 \cdot 0) \leq \max\{\lambda_F(x), \lambda_F(x)\} = \lambda_F(x). \quad \text{((UP-2))}$$

Next, let $x, y, z \in X$. By (3.0.1), we have $(x \cdot (y \cdot z)) \cdot (x \cdot (y \cdot z)) = 0$, that is,

$x \cdot (y \cdot z) \geq x \cdot (y \cdot z)$. It follows from (4.2.24) that

$$\lambda_T(x \cdot z) \leq \max\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\},$$
$$\lambda_I(x \cdot z) \geq \min\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\},$$
$$\lambda_F(x \cdot z) \leq \max\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}.$$

Hence, Λ is a special neutrosophic UP-ideal of X. □

Theorem 4.2.25 *A NS Λ in X satisfies the following condition:*

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} \lambda_T(z) \leq \lambda_T(y) \\ \lambda_I(z) \geq \lambda_I(y) \\ \lambda_F(z) \leq \lambda_F(y) \end{cases}\right) \tag{4.2.25}$$

if and only if Λ is a special neutrosophic strong UP-ideal of X.

Proof. Assume that Λ is a NS in X satisfying the condition (4.2.25). Let $x, y \in X$. By (UP-3) and (3.0.1), we have $x \cdot 0 = 0$, that is, $x \leq 0 = y \cdot y$. It follows from (4.2.25) that $\lambda_T(x) \leq \lambda_T(y), \lambda_I(x) \geq \lambda_I(y)$, and $\lambda_F(x) \leq \lambda_F(y)$. Similarly, $\lambda_T(y) \leq \lambda_T(x), \lambda_I(y) \geq \lambda_I(x)$, and $\lambda_F(y) \leq \lambda_F(x)$. Then $\lambda_T(x) = \lambda_T(y), \lambda_I(x) = \lambda_I(y)$, and $\lambda_F(x) = \lambda_F(y)$. Thus Λ is constant. By Theorem 4.2.17, we have Λ is a special neutrosophic strong UP-ideal of X.

The converse follows from Theorem 4.2.17. □

Then, we have the diagram of generalization of special NSs in UP-algebras as shown in Figure 4.2.

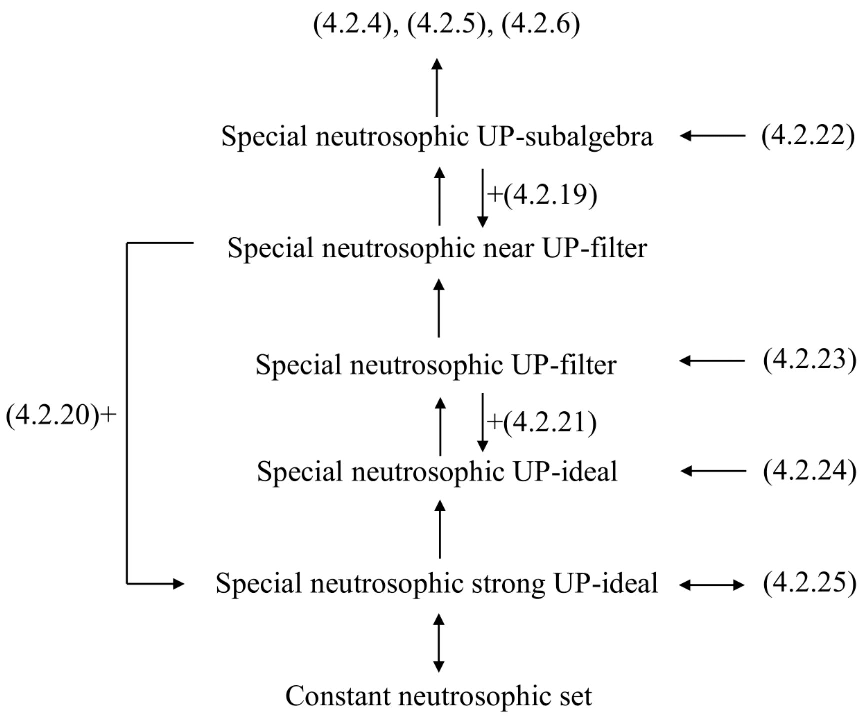

Figure 4.2: Special neutrosophic sets in UP-algebras

For any fixed numbers $\alpha^+, \alpha^-, \beta^+, \beta^-, \gamma^+, \gamma^- \in [0,1]$ such that $\alpha^+ > \alpha^-, \beta^+ > \beta^-, \gamma^+ > \gamma^-$ and a nonempty subset G of X, the NS ${}^G\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}] = (X, {}^G\lambda_T[^{\alpha^-}_{\alpha^+}], {}^G\lambda_I[^{\beta^+}_{\beta^-}], {}^G\lambda_F[^{\gamma^-}_{\gamma^+}])$ in X, where ${}^G\lambda_T[^{\alpha^-}_{\alpha^+}]$, ${}^G\lambda_I[^{\beta^+}_{\beta^-}]$, and ${}^G\lambda_F[^{\gamma^-}_{\gamma^+}]$ are fuzzy sets in X which are given as follows:

$$
{}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x) = \begin{cases} \alpha^- & \text{if } x \in G, \\ \alpha^+ & \text{otherwise,} \end{cases}
$$

$$
{}^G\lambda_I[^{\beta^+}_{\beta^-}](x) = \begin{cases} \beta^+ & \text{if } x \in G, \\ \beta^- & \text{otherwise,} \end{cases}
$$

$$
{}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x) = \begin{cases} \gamma^- & \text{if } x \in G, \\ \gamma^+ & \text{otherwise.} \end{cases}
$$

Lemma 4.2.26 *Let $\alpha^+, \alpha^-, \beta^+, \beta^-, \gamma^+, \gamma^- \in [0,1]$. Then the following statements hold:*

(1) $\overline{\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]} = {}^G\Lambda[^{1-\alpha^+,1-\beta^-,1-\gamma^+}_{1-\alpha^-,1-\beta^+,1-\gamma^-}]$, *and*

(2) $\overline{{}^G\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]} = \Lambda^G[^{1-\alpha^-,1-\beta^+,1-\gamma^-}_{1-\alpha^+,1-\beta^-,1-\gamma^+}]$.

Proof. (1) Let $\overline{\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]}$ be a NS in X. Then $\overline{\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]} = (X, \overline{\lambda^G_T[^{\alpha^+}_{\alpha^-}]}, \overline{\lambda^G_I[^{\beta^-}_{\beta^+}]}, \overline{\lambda^G_F[^{\gamma^+}_{\gamma^-}]})$. Since

$$\lambda^G_T[^{\alpha^+}_{\alpha^-}](x) = \begin{cases} \alpha^+ & \text{if } x \in G, \\ \alpha^- & \text{otherwise,} \end{cases}$$

$$\lambda^G_I[^{\beta^-}_{\beta^+}](x) = \begin{cases} \beta^- & \text{if } x \in G, \\ \beta^+ & \text{otherwise,} \end{cases}$$

$$\lambda^G_F[^{\gamma^+}_{\gamma^-}](x) = \begin{cases} \gamma^+ & \text{if } x \in G, \\ \gamma^- & \text{otherwise.} \end{cases}$$

Thus

$$\overline{\lambda^G_T[^{\alpha^+}_{\alpha^-}]}(x) = \begin{cases} 1-\alpha^+ & \text{if } x \in G, \\ 1-\alpha^- & \text{otherwise} \end{cases} = {}^G\lambda_T[^{1-\alpha^+}_{1-\alpha^-}](x),$$

$$\overline{\lambda^G_I[^{\beta^-}_{\beta^+}]}(x) = \begin{cases} 1-\beta^- & \text{if } x \in G, \\ 1-\beta^+ & \text{otherwise} \end{cases} = {}^G\lambda_I[^{1-\beta^-}_{1-\beta^+}](x),$$

$$\overline{\lambda^G_F[^{\gamma^+}_{\gamma^-}]}(x) = \begin{cases} 1-\gamma^+ & \text{if } x \in G, \\ 1-\gamma^- & \text{otherwise} \end{cases} = {}^G\lambda_F[^{1-\gamma^+}_{1-\gamma^-}](x).$$

Hence, $(X, {}^G\lambda_T[^{1-\alpha^+}_{1-\alpha^-}], {}^G\lambda_I[^{1-\beta^-}_{1-\beta^+}], {}^G\lambda_F[^{1-\gamma^+}_{1-\gamma^-}]) = {}^G\Lambda[^{1-\alpha^+,1-\beta^-,1-\gamma^+}_{1-\alpha^-,1-\beta^+,1-\gamma^-}]$.

(2) Let $\overline{{}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]}$ be a NS in X. Then $\overline{{}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]} = (X, \overline{{}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}]}, \overline{{}^{G}\lambda_I[^{\beta^+}_{\beta^-}]}, \overline{{}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}]})$. Since

$$ {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x) = \begin{cases} \alpha^- & \text{if } x \in G, \\ \alpha^+ & \text{otherwise,} \end{cases} $$

$$ {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x) = \begin{cases} \beta^+ & \text{if } x \in G, \\ \beta^- & \text{otherwise,} \end{cases} $$

$$ {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x) = \begin{cases} \gamma^- & \text{if } x \in G, \\ \gamma^+ & \text{otherwise.} \end{cases} $$

Thus

$$ \overline{{}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}]}(x) = \begin{cases} 1-\alpha^- & \text{if } x \in G, \\ 1-\alpha^+ & \text{otherwise} \end{cases} = \lambda_T^G[^{1-\alpha^-}_{1-\alpha^+}](x), $$

$$ \overline{{}^{G}\lambda_I[^{\beta^+}_{\beta^-}]}(x) = \begin{cases} 1-\beta^+ & \text{if } x \in G, \\ 1-\beta^- & \text{otherwise} \end{cases} = \lambda_I^G[^{1-\beta^+}_{1-\beta^-}](x), $$

$$ \overline{{}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}]}(x) = \begin{cases} 1-\gamma^- & \text{if } x \in G, \\ 1-\gamma^+ & \text{otherwise} \end{cases} = \lambda_F^G[^{1-\gamma^-}_{1-\gamma^+}](x). $$

Hence, $(X, \lambda_T^G[^{1-\alpha^-}_{1-\alpha^+}], \lambda_I^G[^{1-\beta^+}_{1-\beta^-}], \lambda_F^G[^{1-\gamma^-}_{1-\gamma^+}]) = \Lambda^G[^{1-\alpha^-,1-\beta^+,1-\gamma^-}_{1-\alpha^+,1-\beta^-,1-\gamma^+}]$.

$\square$

Lemma 4.2.27 *If the constant* 0 *of* X *is in a nonempty subset* G *of* X*, then a NS* ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ *in* X *satisfies the conditions* (4.2.4), (4.2.5), *and* (4.2.6).

Proof. If $0 \in G$, then ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](0) = \alpha^-$, ${}^{G}\lambda_I[^{\beta^+}_{\beta^-}](0) = \beta^+$, and ${}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](0) = \gamma^-$.

Thus

$$(\forall x \in X)\begin{pmatrix} {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](0) = \alpha^- \leq {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x) \\ {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](0) = \beta^- \geq {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x) \\ {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](0) = \gamma^- \leq {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x) \end{pmatrix}.$$

Hence, ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ satisfies the conditions (4.2.4), (4.2.5), and (4.2.6). □

Lemma 4.2.28 *If a NS* ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ *in* X *satisfies the condition* (4.2.4) *(resp.,* (4.2.5), (4.2.6)*), then the constant* 0 *of* X *is in* G*.*

Proof. Assume that a NS ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ in X satisfies the condition (4.2.4). Then ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](0) \leq {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x)$ for all $x \in X$. Since G is nonempty, there exists $g \in G$. Thus ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](g) = \alpha^-$, so ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](0) \leq {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](g) = \alpha^-$, that is, ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](0) = \alpha^-$. Hence, $0 \in G$. □

Theorem 4.2.29 *A NS* ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ *in* X *is a special neutrosophic UP-subalgebra of* X *if and only if a nonempty subset* G *of* X *is a UP-subalgebra of* X*.*

Proof. Assume that ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a special neutrosophic UP-subalgebra of X. Let $x, y \in G$. Then ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x) = \alpha^- = {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y)$. Thus

$${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y) \leq \max\{{}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x), {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y)\} = \alpha^- \leq {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y) \quad ((4.2.1))$$

and so ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y) = \alpha^-$. Thus $x \cdot y \in G$. Hence, G is a UP-subalgebra of X.

Conversely, assume that G is a UP-subalgebra of X. Let $x, y \in X$.

Case 1: $x, y \in G$. Then

$${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x) = \alpha^- = {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y),$$

$${}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x) = \beta^+ = {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](y),$$

$$^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x) = \gamma^- = {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](y).$$

Thus

$$\max\{^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x), {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y)\} = \alpha^-,$$
$$\min\{^{G}\lambda_I[^{\beta^+}_{\beta^-}](x), {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](y)\} = \beta^+,$$
$$\max\{^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x), {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](y)\} = \gamma^-.$$

Since G is a UP-subalgebra of X, we have $x \cdot y \in G$ and so ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y) = \alpha^-, {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x \cdot y) = \beta^+$, and ${}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x \cdot y) = \gamma^-$. Hence,

$${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y) = \alpha^- \leq \alpha^- = \max\{^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x), {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y)\},$$
$${}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x \cdot y) = \beta^+ \geq \beta^+ = \min\{^{G}\lambda_I[^{\beta^+}_{\beta^-}](x), {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](y)\},$$
$${}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x \cdot y) = \gamma^- \leq \gamma^- = \max\{^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x), {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](y)\}.$$

Case 2: $x \notin G$ or $y \notin G$. Then

$${}^{G}\lambda_T[^{\alpha^+}_{\alpha^-}](x) = \alpha^- \text{ or } {}^{G}\lambda_T[^{\alpha^+}_{\alpha^-}](y) = \alpha^-,$$
$${}^{G}\lambda_I[^{\beta^-}_{\beta^+}](x) = \beta^+ \text{ or } {}^{G}\lambda_I[^{\beta^-}_{\beta^+}](y) = \beta^+,$$
$${}^{G}\lambda_F[^{\gamma^+}_{\gamma^-}](x) = \gamma^- \text{ or } {}^{G}\lambda_F[^{\gamma^+}_{\gamma^-}](y) = \gamma^-.$$

Thus

$$\max\{^{G}\lambda_T[^{\alpha^+}_{\alpha^-}](x), {}^{G}\lambda_T[^{\alpha^+}_{\alpha^-}](y)\} = \alpha^-,$$
$$\min\{^{G}\lambda_I[^{\beta^-}_{\beta^+}](x), {}^{G}\lambda_I[^{\beta^-}_{\beta^+}](y)\} = \beta^+,$$
$$\max\{^{G}\lambda_F[^{\gamma^+}_{\gamma^-}](x), {}^{G}\lambda_F[^{\gamma^+}_{\gamma^-}](y)\} = \gamma^-.$$

Therefore,

$$
\begin{aligned}
{}^{G}\lambda_T[^{\alpha^+}_{\alpha^-}](x\cdot y) \geq \alpha^- &= \max\{{}^{G}\lambda_T[^{\alpha^+}_{\alpha^-}](x), {}^{G}\lambda_T[^{\alpha^+}_{\alpha^-}](y)\},\\
{}^{G}\lambda_I[^{\beta^-}_{\beta^+}](x\cdot y) \leq \beta^+ &= \min\{{}^{G}\lambda_I[^{\beta^-}_{\beta^+}](x), {}^{G}\lambda_I[^{\beta^-}_{\beta^+}](y)\},\\
{}^{G}\lambda_F[^{\gamma^+}_{\gamma^-}](x\cdot y) \geq \gamma^- &= \max\{{}^{G}\lambda_F[^{\gamma^+}_{\gamma^-}](x), {}^{G}\lambda_F[^{\gamma^+}_{\gamma^-}](y)\}.
\end{aligned}
$$

Hence, ${}^{G}\Lambda[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a special neutrosophic UP-subalgebra of X. □

Theorem 4.2.30 *A NS* ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ *in* X *is a special neutrosophic near UP-filter of* X *if and only if a nonempty subset* G *of* X *is a near UP-filter of* X*.*

Proof. Assume that ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a special neutrosophic near UP-filter of X. Since ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ satisfies the condition (4.2.4), it follows from Lemma 4.2.28 that $0 \in G$. Next, let $x \in X$ and $y \in G$. Then ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y) = \alpha^-$. Thus

$$
{}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x\cdot y) \leq {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y) = \alpha^- \leq {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x\cdot y) \tag{(4.2.7)}
$$

and so ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x\cdot y) = \alpha^-$. Thus $x\cdot y \in G$. Hence, G is a near UP-filter of X.

Conversely, assume that G is a near UP-filter of X. Since $0 \in G$, it follows from Lemma 4.2.27 that ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ satisfies the conditions (4.2.4), (4.2.5), and (4.2.6). Next, let $x, y \in X$.

Case 1: $y \in G$. Then ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y) = \alpha^-$, ${}^{G}\lambda_I[^{\beta^+}_{\beta^-}](y) = \beta^+$, and ${}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](y) = \gamma^-$. Since G is a near UP-filter of X, we have $x\cdot y \in G$ and so ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x\cdot y) = \alpha^-$, ${}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x\cdot y) = \beta^+$, and ${}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x\cdot y) = \gamma^-$. Thus

$$
\begin{aligned}
{}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x\cdot y) = \alpha^- \leq \alpha^- &= {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y),\\
{}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x\cdot y) = \beta^+ \geq \beta^+ &= {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](y),\\
{}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x\cdot y) = \gamma^- \leq \gamma^- &= {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](y).
\end{aligned}
$$

Case 2: $y \notin G$. Then ${}^G\lambda_T[^{\alpha^-}_{\alpha^+}](y) = \alpha^+$, ${}^G\lambda_I[^{\beta^+}_{\beta^-}](y) = \beta^-$, and ${}^G\lambda_F[^{\gamma^-}_{\gamma^+}](y)$ $= \gamma^+$. Thus

$$\begin{aligned} {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y) &\leq \alpha^+ = {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](y), \\ {}^G\lambda_I[^{\beta^+}_{\beta^-}](x \cdot y) &\geq \beta^- = {}^G\lambda_I[^{\beta^+}_{\beta^-}](y), \\ {}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x \cdot y) &\leq \gamma^+ = {}^G\lambda_F[^{\gamma^-}_{\gamma^+}](y). \end{aligned}$$

Hence, ${}^G\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a special neutrosophic near UP-filter of X. □

Theorem 4.2.31 *A NS* ${}^G\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ *in* X *is a special neutrosophic UP-filter of* X *if and only if a nonempty subset* G *of* X *is a UP-filter of* X.

Proof. Assume that ${}^G\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a special neutrosophic UP-filter of X. Since ${}^G\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ satisfies the condition (4.2.4), it follows from Lemma 4.2.28 that $0 \in G$. Next, let $x, y \in X$ be such that $x \cdot y \in G$ and $x \in G$. Then ${}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y) = \alpha^- = {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x)$. Thus

$$ {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](y) \leq \max\{{}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y), {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x)\} = \alpha^- \leq {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](y) \qquad ((4.2.10)) $$

and so ${}^G\lambda_T[^{\alpha^-}_{\alpha^+}](y) = \alpha^-$. Thus $y \in G$. Hence, G is a UP-filter of X.

Conversely, assume that G is a UP-filter of X. Since $0 \in G$, it follows from Lemma 4.2.27 that ${}^G\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ satisfies the conditions (4.2.4), (4.2.5), and (4.2.6). Next, let $x, y \in X$.

Case 1: $x \cdot y \in G$ and $x \in G$. Then

$$\begin{aligned} {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y) &= \alpha^- = {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x), \\ {}^G\lambda_I[^{\beta^+}_{\beta^-}](x \cdot y) &= \beta^+ = {}^G\lambda_I[^{\beta^+}_{\beta^-}](x), \\ {}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x \cdot y) &= \gamma^- = {}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x). \end{aligned}$$

Since G is a UP-filter of X, we have $y \in G$ and so ${}^G\lambda_T[^{\alpha^-}_{\alpha^+}](y) = \alpha^-$, ${}^G\lambda_I[^{\beta^+}_{\beta^-}](y) = \beta^+$, and ${}^G\lambda_F[^{\gamma^-}_{\gamma^+}](y) = \gamma^-$. Thus

$$
\begin{aligned}
{}^G\lambda_T[^{\alpha^-}_{\alpha^+}](y) = \alpha^- \leq \alpha^- &= \max\{{}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y), {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x)\}, \\
{}^G\lambda_I[^{\beta^+}_{\beta^-}](y) = \beta^+ \geq \beta^+ &= \min\{{}^G\lambda_I[^{\beta^+}_{\beta^-}](x \cdot y), {}^G\lambda_I[^{\beta^+}_{\beta^-}](x)\}, \\
{}^G\lambda_F[^{\gamma^-}_{\gamma^+}](y) = \gamma^- \leq \gamma^+ &= \max\{{}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x \cdot y), {}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x)\}.
\end{aligned}
$$

Case 2: $x \cdot y \notin G$ or $x \notin G$. Then

$$
\begin{aligned}
{}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y) = \alpha^+ &\text{ or } {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x) = \alpha^+, \\
{}^G\lambda_I[^{\beta^+}_{\beta^-}](x \cdot y) = \beta^- &\text{ or } {}^G\lambda_I[^{\beta^+}_{\beta^-}](x) = \beta^-, \\
{}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x \cdot y) = \gamma^+ &\text{ or } {}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x) = \gamma^+.
\end{aligned}
$$

Thus

$$
\begin{aligned}
\max\{{}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y), {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x)\} &= \alpha^+, \\
\min\{{}^G\lambda_I[^{\beta^+}_{\beta^-}](x \cdot y), {}^G\lambda_I[^{\beta^+}_{\beta^-}](x)\} &= \beta^-, \\
\max\{{}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x \cdot y), {}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x)\} &= \gamma^+.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
{}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x) \leq \alpha^+ &= \max\{{}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y), {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x)\}, \\
{}^G\lambda_I[^{\beta^+}_{\beta^-}](x) \geq \beta^- &= \min\{{}^G\lambda_I[^{\beta^+}_{\beta^-}](x \cdot y), {}^G\lambda_I[^{\beta^+}_{\beta^-}](x)\}, \\
{}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x) \leq \gamma^+ &= \max\{{}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x \cdot y), {}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x)\}.
\end{aligned}
$$

Hence, ${}^G\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a special neutrosophic UP-filter of X. □

Theorem 4.2.32 *A NS ${}^G\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ in X is a special neutrosophic UP-ideal of X if and only if a nonempty subset G of X is a UP-ideal of X.*

Proof. Assume that ${}^G\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a special neutrosophic UP-ideal of X. Since ${}^G\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ satisfies the condition (4.2.4), it follows from Lemma 4.2.28 that $0 \in G$. Next, let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in G$ and $y \in G$. Then ${}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot (y \cdot z)) = \alpha^- = {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](y)$. Thus

$$ {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot z) \leq \max\{{}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot (y \cdot z)), {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](y)\} = \alpha^- \leq {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot z) \tag{(4.2.13)} $$

and so ${}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot z) = \alpha^-$. Thus $x \cdot z \in G$. Hence, G is a UP-ideal of X.

Conversely, assume that G is a UP-ideal of X. Since $0 \in G$, it follows from Lemma 4.2.27 that ${}^G\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ satisfies the conditions (4.2.4), (4.2.5), and (4.2.6). Next, let $x, y, z \in X$.

Case 1: $x \cdot (y \cdot z) \in G$ and $y \in G$. Then

$$ \begin{aligned} {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot (y \cdot z)) &= \alpha^- = {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](y), \\ {}^G\lambda_I[^{\beta^+}_{\beta^-}](x \cdot (y \cdot z)) &= \beta^+ = {}^G\lambda_I[^{\beta^+}_{\beta^-}](y), \\ {}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x \cdot (y \cdot z)) &= \gamma- = {}^G\lambda_F[^{\gamma^-}_{\gamma^+}](y). \end{aligned} $$

Thus

$$ \begin{aligned} \max\{{}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot (y \cdot z)), {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](y)\} &= \alpha^-, \\ \min\{{}^G\lambda_I[^{\beta^+}_{\beta^-}](x \cdot (y \cdot z)), {}^G\lambda_I[^{\beta^+}_{\beta^-}](y)\} &= \beta^+, \\ \max\{{}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x \cdot (y \cdot z)), {}^G\lambda_F[^{\gamma^-}_{\gamma^+}](y)\} &= \gamma^-. \end{aligned} $$

Since G is a UP-ideal of X, we have $x \cdot z \in G$ and so ${}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot z) = \alpha^-$, ${}^G\lambda_I[^{\beta^+}_{\beta^-}](x \cdot z) = \beta^+$, and ${}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x \cdot z) = \gamma^-$. Thus

$$ {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot z) = \alpha^- \leq \alpha^- = \max\{{}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot (y \cdot z)), {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](y)\}, $$

$$^{G}\lambda_I[^{\beta^+}_{\beta^-}](x\cdot z)=\beta^+\geq\beta^+=\min\{^{G}\lambda_I[^{\beta^+}_{\beta^-}](x\cdot(y\cdot z)),{}^{G}\lambda_I[^{\beta^+}_{\beta^-}](y)\},$$
$$^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x\cdot z)=\gamma^-\leq\gamma^-=\max\{^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x\cdot(y\cdot z)),{}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](y)\}.$$

Case 2: $x\cdot(y\cdot z)\notin G$ or $y\notin G$. Then

$$^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x\cdot(y\cdot z))=\alpha^+ \text{ or } {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y)=\alpha^+,$$
$$^{G}\lambda_I[^{\beta^+}_{\beta^-}](x\cdot(y\cdot z))=\beta^- \text{ or } {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](y)=\beta^-,$$
$$^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x\cdot(y\cdot z))=\gamma^+ \text{ or } {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](y)=\gamma^+.$$

Thus

$$\max\{^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x\cdot(y\cdot z)),{}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y)\}=\alpha^+,$$
$$\min\{^{G}\lambda_I[^{\beta^+}_{\beta^-}](x\cdot(y\cdot z)),{}^{G}\lambda_I[^{\beta^+}_{\beta^-}](y)\}=\beta^-,$$
$$\max\{^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x\cdot(y\cdot z)),{}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](y)\}=\gamma^+.$$

Therefore,

$$^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x\cdot z)\leq\alpha^+=\max\{^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x\cdot(y\cdot z)),{}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y)\},$$
$$^{G}\lambda_I[^{\beta^+}_{\beta^-}](x\cdot z)\geq\beta^-=\min\{^{G}\lambda_I[^{\beta^+}_{\beta^-}](x\cdot(y\cdot z)),{}^{G}\lambda_I[^{\beta^+}_{\beta^-}](y)\},$$
$$^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x\cdot z)\leq\gamma^+=\max\{^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x\cdot(y\cdot z)),{}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](y)\}.$$

Hence, $^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a special neutrosophic UP-ideal of X. □

Theorem 4.2.33 *A NS $^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ in X is a special neutrosophic strong UP-ideal of X if and only if a nonempty subset G of X is a strong UP-ideal of X.*

Proof. Assume that $^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a special neutrosophic strong UP-ideal of X. By Theorem 4.2.17, we have $^{G}\lambda_T[^{\alpha^-}_{\alpha^+}]$ is constant, that is, $^{G}\lambda_T[^{\alpha^-}_{\alpha^+}]$ is constant. Since G is nonempty, we have $^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x)=\alpha^-$ for all $x\in X$. Thus $G=X$.

Hence, G is a strong UP-ideal of X.

Conversely, assume that G is a strong UP-ideal of X. Then $G = X$, so

$$(\forall x \in X)\left(\begin{array}{c} {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x) = \alpha^- \\ {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x) = \beta^+ \\ {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x) = \gamma^- \end{array}\right).$$

Thus ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}]$, ${}^{G}\lambda_I[^{\beta^+}_{\beta^-}]$, and ${}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}]$ are constant, that is, ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is constant. By Theorem 4.2.17, we have ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a special neutrosophic strong UP-ideal of X. □

Next, we discuss the relationships among special neutrosophic UP-subalgebras (resp., special neutrosophic near UP-filters, special neutrosophic UP-filters, special neutrosophic UP-ideals, special neutrosophic strong UP-ideals) of UP-algebras and their level subsets.

Theorem 4.2.34 *A NS Λ in X is a special neutrosophic UP-subalgebra of X if and only if for all $\alpha, \beta, \gamma \in [0,1]$, the sets $L(\lambda_T;\alpha), U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are either empty or UP-subalgebras of X.*

Proof. Assume that Λ is a special neutrosophic UP-subalgebra of X. Let $\alpha, \beta, \gamma \in [0,1]$ be such that $L(\lambda_T;\alpha), U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are nonempty.

Let $x, y \in L(\lambda_T;\alpha)$. Then $\lambda_T(x) \leq \alpha$ and $\lambda_T(y) \leq \alpha$, so α is a upper bound of $\{\lambda_T(x), \lambda_T(y)\}$. By (4.2.1), we have $\lambda_T(x\cdot y) \leq \max\{\lambda_T(x), \lambda_T(y)\} \leq \alpha$. Thus $x \cdot y \in L(\lambda_T;\alpha)$.

Let $x, y \in U(\lambda_I;\beta)$. Then $\lambda_I(x) \geq \beta$ and $\lambda_I(y) \geq \beta$, so β is an lower bound of $\{\lambda_I(x), \lambda_I(y)\}$. By (4.2.2), we have $\lambda_I(x \cdot y) \geq \min\{\lambda_I(x), \lambda_I(y)\} \geq \beta$. Thus $x \cdot y \in U(\lambda_I;\beta)$.

Let $x, y \in L(\lambda_F; \gamma)$. Then $\lambda_F(x) \leq \gamma$ and $\lambda_F(y) \leq \gamma$, so γ is a upper bound of $\{\lambda_F(x), \lambda_F(y)\}$. By (4.2.3), we have $\lambda_F(x \cdot y) \leq \max\{\lambda_F(x), \lambda_F(y)\} \leq \gamma$. Thus $x \cdot y \in L(\lambda_F; \gamma)$.

Hence, $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are UP-subalgebras of X.

Conversely, assume that for all $\alpha, \beta, \gamma \in [0, 1]$, the set $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are UP-subalgebras if $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are nonempty.

Let $x, y \in X$. Then $\lambda_T(x), \lambda_T(y) \in [0, 1]$. Choose $\alpha = \max\{\lambda_T(x), \lambda_T(y)\}$. Thus $\lambda_T(x) \leq \alpha$ and $\lambda_T(y) \leq \alpha$, so $x, y \in L(\lambda_T; \alpha) \neq \emptyset$. By assumption, we have $L(\lambda_T; \alpha)$ is a UP-subalgebra of X and so $x, y \in L(\lambda_T; \alpha)$. Thus $\lambda_T(x \cdot y) \leq \alpha = \max\{\lambda_T(x), \lambda_T(y)\}$.

Let $x, y \in X$. Then $\lambda_I(x), \lambda_I(y) \in [0, 1]$. Choose $\beta = \min\{\lambda_I(x), \lambda_I(y)\}$. Thus $\lambda_I(x) \geq \beta$ and $\lambda_I(y) \geq \beta$, so $x, y \in U(\lambda_I; \beta) \neq \emptyset$. By assumption, we have $U(\lambda_I; \beta)$ is a UP-subalgebra of X and so $x, y \in U(\lambda_I; \beta)$. Thus $\lambda_I(x \cdot y) \geq \beta = \min\{\lambda_I(x), \lambda_I(y)\}$.

Let $x, y \in X$. Then $\lambda_F(x), \lambda_F(y) \in [0, 1]$. Choose $\gamma = \max\{\lambda_F(x), \lambda_F(y)\}$. Thus $\lambda_F(x) \leq \gamma$ and $\lambda_F(y) \leq \gamma$, so $x, y \in L(\lambda_F; \gamma) \neq \emptyset$. By assumption, we have $L(\lambda_F; \gamma)$ is a UP-subalgebra of X and so $x, y \in L(\lambda_F; \gamma)$. Thus $\lambda_F(x \cdot y) \leq \gamma = \max\{\lambda_F(x), \lambda_F(y)\}$.

Therefore, Λ is a special neutrosophic UP-subalgebra of X. □

Theorem 4.2.35 *A NS Λ in X is a special neutrosophic near UP-filter of X if and only if for all $\alpha, \beta, \gamma \in [0, 1]$, the sets $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are either empty or near UP-filters of X.*

Proof. Assume that Λ is a special neutrosophic near UP-filter of X. Let $\alpha, \beta, \gamma \in [0, 1]$ be such that $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are nonempty.

Let $x \in L(\lambda_T;\alpha)$. Then $\lambda_T(x) \leq \alpha$. By (4.2.4), we have $\lambda_T(0) \leq \lambda_T(x) \leq \alpha$. Thus $0 \in L(\lambda_T;\alpha)$. Next, let $y \in L(\lambda_T;\alpha)$. Then $\lambda_T(y) \leq \alpha$. By (4.2.7), we have $\lambda_T(x \cdot y) \leq \lambda_T(y) \leq \alpha$. Thus $x \cdot y \in L(\lambda_T;\alpha)$.

Let $x \in U(\lambda_I;\beta)$. Then $\lambda_I(x) \geq \beta$. By (4.2.5), we have $\lambda_I(0) \geq \lambda_I(x) \geq \beta$. Thus $0 \in U(\lambda_I;\beta)$. Next, let $y \in U(\lambda_I;\beta)$. Then $\lambda_I(y) \geq \beta$. By (4.2.8), we have $\lambda_I(x \cdot y) \geq \lambda_I(y) \geq \beta$. Thus $x \cdot y \in U(\lambda_I;\beta)$.

Let $x \in L(\lambda_F;\gamma)$. Then $\lambda_F(x) \leq \gamma$. By (4.2.6), we have $\lambda_F(0) \leq \lambda_F(x) \leq \gamma$. Thus $0 \in L(\lambda_F;\gamma)$. Next, $y \in L(\lambda_F;\gamma)$. Then $\lambda_F(y) \leq \gamma$. By (4.2.8), we have $\lambda_F(x \cdot y) \leq \lambda_F(y) \leq \gamma$. Thus $x \cdot y \in L(\lambda_F;\gamma)$.

Hence, $L(\lambda_T;\alpha)$, $U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are near UP-filters of X.

Conversely, assume that for all $\alpha, \beta, \gamma \in [0,1]$, the set $L(\lambda_T;\alpha), U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are near UP-filters if $L(\lambda_T;\alpha), U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are nonempty.

Let $x \in X$. Then $\lambda_T(0) \in [0,1]$. Choose $\alpha = \lambda_T(x)$. Thus $\lambda_T(x) \leq \alpha$, so $x \in L(\lambda_T;\alpha) \neq \emptyset$. By assumption, we have $L(\lambda_T;\alpha)$ is a near UP-filter of X and so $0 \in L(\lambda_T;\alpha)$. Thus $\lambda_T(0) \leq \alpha = \lambda_T(x)$. Next, let $y \in X$. Then $\lambda_T(y) \in [0,1]$. Choose $\alpha = \lambda_T(y)$. Thus $\lambda_T(y) \leq \alpha$, so $y \in L(\lambda_T;\alpha) \neq \emptyset$. By assumption, we have $L(\lambda_T;\alpha)$ is a near UP-filter of X, and so $x \cdot y \in L(\lambda_T;\alpha)$. Thus $\lambda_T(x \cdot y) \leq \alpha = \lambda_T(y)$.

Let $x \in X$. Then $\lambda_I(0) \in [0,1]$. Choose $\beta = \lambda_I(x)$. Thus $\lambda_I(x) \geq \beta$, so $x \in U(\lambda_I;\beta) \neq \emptyset$. By assumption, we have $U(\lambda_I;\beta)$ is a near UP-filter of X and so $0 \in U(\lambda_I;\beta)$. Thus $\lambda_I(0) \geq \beta = \lambda_I(x)$. Next, let $y \in X$. Then $\lambda_I(y) \in [0,1]$. Choose $\beta = \lambda_I(y)$. Thus $\lambda_I(y) \geq \beta$, so $y \in U(\lambda_I;\beta) \neq \emptyset$. By assumption, we have $U(\lambda_I;\beta)$ is a near UP-filter of X, and so $x \cdot y \in U(\lambda_I;\beta)$. Thus $\lambda_I(x \cdot y) \geq \beta = \lambda_I(y)$.

Let $x \in X$. Then $\lambda_F(0) \in [0,1]$. Choose $\gamma = \lambda_F(x)$. Thus $\lambda_F(x) \leq \gamma$, so $x \in L(\lambda_F; \gamma) \neq \emptyset$. By assumption, we have $L(\lambda_F; \gamma)$ is a near UP-filter of X and so $0 \in L(\lambda_F; \gamma)$. Thus $\lambda_F(0) \leq \gamma = \lambda_F(x)$. Next, let $y \in X$. Then $\lambda_F(y) \in [0, 1]$. Choose $\gamma = \lambda_F(y)$. Thus $\lambda_F(y) \leq \gamma$, so $y \in L(\lambda_F; \gamma) \neq \emptyset$. By assumption, we have $L(\lambda_F; \gamma)$ is a near UP-filter of X, and so $x \cdot y \in L(\lambda_F; \gamma)$. Thus $\lambda_F(x \cdot y) \leq \gamma = \lambda_F(y)$.

Therefore, Λ is a special neutrosophic near UP-filter of X. □

Theorem 4.2.36 *A NS Λ in X is a special neutrosophic UP-filter of X if and only if for all $\alpha, \beta, \gamma \in [0, 1]$, the sets $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are either empty or UP-filters of X.*

Proof. Assume that Λ is a special neutrosophic UP-filter of X. Let $\alpha, \beta, \gamma \in [0, 1]$ be such that $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are nonempty.

Let $x \in L(\lambda_T; \alpha)$. Then $\lambda_T(x) \leq \alpha$. By (4.2.4), we have $\lambda_T(0) \leq \lambda_T(x) \leq \alpha$. Thus $0 \in L(\lambda_T; \alpha)$. Next, let $x \cdot y \in L(\lambda_T; \alpha)$ and $x \in L(\lambda_T; \alpha)$. Then $\lambda_T(x \cdot y) \leq \alpha$ and $\lambda_T(x) \leq \alpha$, so α is a upper bound of $\{\lambda_T(x \cdot y), \lambda_T(x)\}$. By (4.2.10), we have $\lambda_T(y) \leq \max\{\lambda_T(x \cdot y), \lambda_T(x)\} \leq \alpha$. Thus $y \in L(\lambda_T; \alpha)$.

Let $x \in U(\lambda_I; \beta)$. Then $\lambda_I(x) \geq \beta$. By (4.2.5), we have $\lambda_I(0) \geq \lambda_I(x) \geq \beta$. Thus $0 \in U(\lambda_I; \beta)$. Next, let $x \cdot y \in U(\lambda_I; \beta)$ and $x \in U(\lambda_I; \beta)$. Then $\lambda_I(x \cdot y) \geq \beta$ and $\lambda_I(x) \geq \beta$, so β is an lower bound of $\{\lambda_I(x \cdot y), \lambda_I(x)\}$. By (4.2.11), we have $\lambda_I(y) \geq \min\{\lambda_I(x \cdot y), \lambda_I(x)\} \geq \beta$. Thus $y \in U(\lambda_I; \beta)$.

Let $x \in L(\lambda_F; \gamma)$. Then $\lambda_F(x) \leq \gamma$. By (4.2.6), we have $\lambda_F(0) \leq \lambda_F(x) \leq \gamma$. Thus $0 \in L(\lambda_F; \gamma)$. Next, let $x \cdot y \in L(\lambda_F; \gamma)$ and $x \in L(\lambda_F; \gamma)$. Then $\lambda_F(x \cdot y) \leq \gamma$ and $\lambda_F(x) \leq \gamma$, so γ is a upper bound of $\{\lambda_F(x \cdot y), \lambda_F(x)\}$. By (4.2.12), we have $\lambda_F(y) \leq \max\{\lambda_F(x \cdot y), \lambda_F(x)\} \leq \gamma$. Thus $y \in L(\lambda_F; \gamma)$.

Hence, $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are UP-filters of X.

Conversely, assume that for all $\alpha, \beta, \gamma \in [0,1]$, the set $L(\lambda_T;\alpha), U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are UP-filters if $L(\lambda_T;\alpha), U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are nonempty.

Let $x \in X$. Then $\lambda_T(x) \in [0,1]$. Choose $\alpha = \lambda_T(x)$. Thus $\lambda_T(x) \leq \alpha$, so $x \in L(\lambda_T;\alpha) \neq \emptyset$. By assumption, we have $L(\lambda_T;\alpha)$ is a UP-filter of X and so $0 \in L(\lambda_T;\alpha)$. Thus $\lambda_T(0) \leq \alpha = \lambda_T(x)$. Next, let $x, y \in X$. Then $\lambda_T(x \cdot y), \lambda_T(x) \in [0,1]$. Choose $\alpha = \max\{\lambda_T(x \cdot y), \lambda_T(x)\}$. Thus $\lambda_T(x \cdot y) \leq \alpha$ and $\lambda_T(x) \leq \alpha$, so $x \cdot y, x \in L(\lambda_T;\alpha) \neq \emptyset$. By assumption, we have $L(\lambda_T;\alpha)$ is a UP-filter of X and so $y \in L(\lambda_T;\alpha)$. Thus $\lambda_T(y) \leq \alpha = \max\{\lambda_T(x \cdot y), \lambda_T(x)\}$.

Let $x \in X$. Then $\lambda_I(x) \in [0,1]$. Choose $\beta = \lambda_I(x)$. Thus $\lambda_I(x) \geq \beta$, so $x \in U(\lambda_I;\beta) \neq \emptyset$. By assumption, we have $U(\lambda_I;\beta)$ is a UP-filter of X and so $0 \in U(\lambda_I;\beta)$. Thus $\lambda_I(0) \geq \beta = \lambda_I(x)$. Next, let $x, y \in X$. Then $\lambda_I(x \cdot y), \lambda_I(x) \in [0,1]$. Choose $\beta = \min\{\lambda_I(x \cdot y), \lambda_I(x)\}$. Thus $\lambda_I(x \cdot y) \geq \beta$ and $\lambda_I(x) \geq \beta$, so $x \cdot y, x \in U(\lambda_I;\beta) \neq \emptyset$. By assumption, we have $U(\lambda_I;\beta)$ is a UP-filter of X and so $y \in U(\lambda_I;\beta)$. Thus $\lambda_I(y) \geq \beta = \min\{\lambda_I(x \cdot y), \lambda_I(x)\}$.

Let $x \in X$. Then $\lambda_F(x) \in [0,1]$. Choose $\gamma = \lambda_F(x)$. Thus $\lambda_F(x) \leq \gamma$, so $x \in L(\lambda_F;\gamma) \neq \emptyset$. By assumption, we have $L(\lambda_F;\gamma)$ is a UP-filter of X and so $0 \in L(\lambda_F;\gamma)$. Thus $\lambda_F(0) \leq \gamma = \lambda_F(x)$. Next, let $x, y \in X$. Then $\lambda_F(x \cdot y), \lambda_F(x) \in [0,1]$. Choose $\gamma = \max\{\lambda_F(x \cdot y), \lambda_F(x)\}$. Thus $\lambda_F(x \cdot y) \leq \gamma$ and $\lambda_F(x) \leq \gamma$, so $x \cdot y, x \in L(\lambda_F;\gamma) \neq \emptyset$. By assumption, we have $L(\lambda_F;\gamma)$ is a UP-filter of X and so $y \in L(\lambda_F;\gamma)$. Thus $\lambda_F(y) \leq \gamma = \max\{\lambda_F(x \cdot y), \lambda_F(x)\}$.

Therefore, Λ is a special neutrosophic UP-filter of X. □

Theorem 4.2.37 *A NS Λ in X is a special neutrosophic UP-ideals of X if and only if for all $\alpha, \beta, \gamma \in [0,1]$, the sets $L(\lambda_T;\alpha), U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are either empty or UP-ideals of X.*

Proof. Assume that Λ is a special neutrosophic UP-ideal of X. Let $\alpha, \beta, \gamma \in [0,1]$ be such that $L(\lambda_T;\alpha), U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are nonempty.

Let $x \in L(\lambda_T;\alpha)$. Then $\lambda_T(x) \leq \alpha$. By (4.2.4), we have $\lambda_T(0) \leq \lambda_T(x) \leq \alpha$. Thus $0 \in L(\lambda_T;\alpha)$. Next, let $x \cdot (y \cdot z) \in L(\lambda_T;\alpha)$ and $y \in L(\lambda_T;\alpha)$. Then $\lambda_T(x\cdot(y\cdot z)) \leq \alpha$ and $\lambda_T(y) \leq \alpha$, so α is a upper bound of $\{\lambda_T(x\cdot(y\cdot z)), \lambda_T(y)\}$. By (4.2.13), we have $\lambda_T(x\cdot z) \leq \max\{\lambda_T(x\cdot(y\cdot z)), \lambda_T(y)\} \leq \alpha$. Thus $x\cdot z \in L(\lambda_T;\alpha)$.

Let $x \in U(\lambda_I;\beta)$. Then $\lambda_I(x) \geq \beta$. By (4.2.5), we have $\lambda_I(0) \geq \lambda_I(x) \geq \beta$. Thus $0 \in U(\lambda_I;\beta)$. Next, let $x \cdot (y \cdot z) \in U(\lambda_I;\beta)$ and $y \in U(\lambda_I;\beta)$. Then $\lambda_I(x\cdot(y\cdot z)) \geq \beta$ and $\lambda_I(y) \geq \beta$, so β is an lower bound of $\{\lambda_I(x\cdot(y\cdot z)), \lambda_I(y)\}$. By (4.2.14), we have $\lambda_I(x\cdot z) \geq \min\{\lambda_I(x\cdot(y\cdot z)), \lambda_I(y)\} \geq \beta$. Thus $x\cdot z \in U(\lambda_I;\beta)$.

Let $x \in L(\lambda_F;\gamma)$. Then $\lambda_F(x) \leq \gamma$. By (4.2.6), we have $\lambda_F(0) \leq \lambda_F(x) \leq \gamma$. Thus $0 \in L(\lambda_F;\gamma)$. Next, let $x \cdot (y \cdot z) \in L(\lambda_F;\gamma)$ and $y \in L(\lambda_F;\gamma)$. Then $\lambda_F(x\cdot(y\cdot z)) \leq \gamma$ and $\lambda_F(y) \leq \gamma$, so γ is a upper bound of $\{\lambda_F(x\cdot(y\cdot z)), \lambda_F(y)\}$. By (4.2.15), we have $\lambda_F(x\cdot z) \leq \max\{\lambda_F(x\cdot(y\cdot z)), \lambda_F(y)\} \leq \gamma$. Thus $x\cdot z \in L(\lambda_F;\gamma)$.

Hence, $L(\lambda_T;\alpha), U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are UP-ideals of X.

Conversely, assume that for all $\alpha, \beta, \gamma \in [0,1]$, the set $L(\lambda_T;\alpha), U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are UP-ideals if $L(\lambda_T;\alpha), U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are nonempty.

Let $x \in X$. Then $\lambda_T(x) \in [0,1]$. Choose $\alpha = \lambda_T(x)$. Thus $\lambda_T(x) \leq \alpha$, so $x \in L(\lambda_T;\alpha) \neq \emptyset$. By assumption, we have $L(\lambda_T;\alpha)$ is a UP-ideal of X and so $0 \in L(\lambda_T;\alpha)$. Thus $\lambda_T(0) \leq \alpha = \lambda_T(x)$. Next, let $x, y, z \in X$. Then $\lambda_T(x \cdot (y \cdot z)), \lambda_T(y) \in [0,1]$. Choose $\alpha = \max\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\}$. Thus $\lambda_T(x \cdot (y \cdot z)) \leq \alpha$ and $\lambda_T(y) \leq \alpha$, so $x \cdot (y \cdot z), y \in L(\lambda_T;\alpha) \neq \emptyset$. By assumption, we have $L(\lambda_T;\alpha)$ is a UP-ideal of X and so $x \cdot z \in L(\lambda_T;\alpha)$. Thus $\lambda_T(x \cdot z) \leq \alpha = \max\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\}$.

Let $x \in X$. Then $\lambda_I(x) \in [0,1]$. Choose $\beta = \lambda_I(x)$. Thus $\lambda_I(x) \geq \beta$, so $x \in U(\lambda_I;\beta) \neq \emptyset$. By assumption, we have $U(\lambda_I;\beta)$ is a UP-ideal of X and so $0 \in U(\lambda_I;\beta)$. Thus $\lambda_I(0) \geq \beta = \lambda_I(x)$. Next, let $x, y, z \in X$. Then $\lambda_I(x \cdot (y \cdot$

$z)), \lambda_I(y) \in [0,1]$. Choose $\beta = \min\{\lambda_I(x\cdot(y\cdot z)), \lambda_I(y)\}$. Thus $\lambda_I(x\cdot(y\cdot z)) \geq \beta$ and $\lambda_I(y) \geq \beta$, so $x\cdot(y\cdot z), y \in U(\lambda_I;\beta) \neq \emptyset$. By assumption, we have $U(\lambda_I;\beta)$ is a UP-ideal of X and so $x \cdot z \in U(\lambda_I;\beta)$. Thus $\lambda_I(x \cdot z) \geq \beta = \min\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\}$.

Let $x \in X$. Then $\lambda_F(x) \in [0,1]$. Choose $\gamma = \lambda_F(x)$. Thus $\lambda_F(x) \leq \gamma$, so $x \in L(\lambda_F;\gamma) \neq \emptyset$. By assumption, we have $L(\lambda_F;\gamma)$ is a UP-ideal of X and so $0 \in L(\lambda_F;\gamma)$. Thus $\lambda_F(0) \leq \gamma = \lambda_F(x)$. Next, let $x, y, z \in X$. Then $\lambda_F(x \cdot (y \cdot z)), \lambda_F(y) \in [0,1]$. Choose $\gamma = \max\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}$. Thus $\lambda_F(x \cdot (y \cdot z)) \leq \gamma$ and $\lambda_F(y) \leq \gamma$, so $x \cdot (y \cdot z), y \in L(\lambda_F;\gamma) \neq \emptyset$. By assumption, we have $L(\lambda_F;\gamma)$ is a UP-ideal of X and so $x \cdot z \in L(\lambda_F;\gamma)$. Thus $\lambda_F(x \cdot z) \leq \gamma = \max\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}$.

Therefore, Λ is a special neutrosophic UP-ideal of X. □

Theorem 4.2.38 *A NS Λ in X is a special neutrosophic strong UP-ideal of X if and only if the sets $E(\lambda_T;\lambda_T(0)), E(\lambda_I;\lambda_I(0))$, and $E(\lambda_F;\lambda_F(0))$ are strong UP-ideals of X.*

Proof. It is straightforward by Theorems 4.1.13, 4.1.41, and 4.2.17. □

Corollary 4.2.39 *A NS Λ in X is a special neutrosophic UP-subalgebra of X if and only if for all $\alpha, \beta, \gamma \in [0,1]$, $LUL_\Lambda(\alpha, \beta, \gamma)$ is a UP-subalgebra of X, where $LUL_\Lambda(\alpha, \beta, \gamma)$ is nonempty.*

Proof. It is straightforward by Theorems 3.0.6 and 4.2.34. □

Corollary 4.2.40 *A NS Λ in X is a special neutrosophic near UP-filter of X if and only if for all $\alpha, \beta, \gamma \in [0,1]$, $LUL_\Lambda(\alpha, \beta, \gamma)$ is a near UP-filter of X, where $LUL_\Lambda(\alpha, \beta, \gamma)$ is nonempty.*

Proof. It is straightforward by Theorems 3.0.6 and 4.2.35. □

Corollary 4.2.41 *A NS* Λ *in* X *is a special neutrosophic UP-filter of* X *if and only if for all* $\alpha, \beta, \gamma \in [0, 1]$, $LUL_\Lambda(\alpha, \beta, \gamma)$ *is a UP-filter of* X, *where* $LUL_\Lambda(\alpha, \beta, \gamma)$ *is nonempty.*

Proof. It is straightforward by Theorems 3.0.6 and 4.2.36. □

Corollary 4.2.42 *A NS* Λ *in* X *is a special neutrosophic UP-ideal of* X *if and only if for all* $\alpha, \beta, \gamma \in [0, 1]$, $LUL_\Lambda(\alpha, \beta, \gamma)$ *is a UP-ideal of* X, *where* $LUL_\Lambda(\alpha, \beta, \gamma)$ *is nonempty.*

Proof. It is straightforward by Theorems 3.0.6 and 4.2.37. □

Corollary 4.2.43 *A NS* Λ *in* X *is a special neutrosophic strong UP-ideal of* X *if and only if* $E_\Lambda(\lambda_T(0), \lambda_I(0), \lambda_F(0))$ *is a strong UP-ideal of* X.

Proof. It is straightforward by Theorems 3.0.6 and 4.2.38. □

4.3 Interval-valued neutrosophic sets in UP-algebras

From closed subinterval of unit interval $[0, 1]$, we introduce the concepts of interval-valued neutrosophic UP-subalgebras, interval-valued neutrosophic near UP-filters, interval-valued neutrosophic UP-filters, interval-valued neutrosophic UP-ideals, and interval-valued neutrosophic strong UP-ideals of UP-algebras, provide the necessary examples, investigate their properties, and prove their generalizations.

Definition 4.3.1 An IVNS $\mathbf{A}$ in X is called an *interval-valued neutrosophic UP-subalgebra* of X if it holds the following conditions:

$$(\forall x, y \in X)(A_T(x \cdot y) \succeq \text{rmin}\{A_T(x), A_T(y)\}), \tag{4.3.1}$$

$$(\forall x, y \in X)(A_I(x \cdot y) \preceq \text{rmax}\{A_I(x), A_I(y)\}), \text{ and} \tag{4.3.2}$$

$$(\forall x, y \in X)(A_F(x \cdot y) \succeq \text{rmin}\{A_F(x), A_F(y)\}). \tag{4.3.3}$$

Proposition 4.3.2 *If* **A** *is an interval-valued neutrosophic UP-subalgebra of* X*, then*

$$(\forall x \in X)(A_T(0) \succeq A_T(x)), \tag{4.3.4}$$

$$(\forall x \in X)(A_I(0) \preceq A_I(x)), \text{ and} \tag{4.3.5}$$

$$(\forall x \in X)(A_F(0) \succeq A_F(x)). \tag{4.3.6}$$

Proof. Let **A** be an interval-valued neutrosophic UP-subalgebra of X. By (3.0.1), we have

$$(\forall x \in X)\begin{pmatrix} A_T(0) = A_T(x \cdot x) \succeq \text{rmin}\{A_T(x), A_T(x)\} = A_T(x), \\ A_I(0) = A_I(x \cdot x) \preceq \text{rmin}\{A_I(x), A_I(x)\} = A_I(x), \text{ and} \\ A_F(0) = A_F(x \cdot x) \succeq \text{rmin}\{A_F(x), A_F(x)\} = A_F(x) \end{pmatrix}.$$

□

Example 4.3.3 Let $X = \{0, 1, 2, 3\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	0	2
2	0	1	0	3
3	0	0	0	0

We define an IVNS **A** in X as follows:

$$A_T = \begin{pmatrix} 0 & 1 & 2 & 3 \\ [0.9, 1] & [0.2, 0.5] & [0.3, 0.4] & [0.3, 0.4] \end{pmatrix},$$

$$A_I = \begin{pmatrix} 0 & 1 & 2 & 3 \\ [0, 0.3] & [0.7, 0.8] & [0.2, 0.3] & [0.8, 0.9] \end{pmatrix},$$
$$A_F = \begin{pmatrix} 0 & 1 & 2 & 3 \\ [0.7, 1] & [0.1, 0.3] & [0.5, 0.7] & [0.6, 0.7] \end{pmatrix}.$$

Then **A** is an interval-valued neutrosophic UP-subalgebra of X.

Definition 4.3.4 An IVNS **A** in X is called an *interval-valued neutrosophic near UP-filter* of X if it holds the following conditions: (4.3.4), (4.3.5), (4.3.6),

$$(\forall x, y \in X)(A_T(x \cdot y) \succeq A_T(y)), \tag{4.3.7}$$

$$(\forall x, y \in X)(A_I(x \cdot y) \preceq A_I(y)), \text{ and} \tag{4.3.8}$$

$$(\forall x, y \in X)(A_F(x \cdot y) \succeq A_F(y)). \tag{4.3.9}$$

Example 4.3.5 Let $X = \{0, 1, 2, 3\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	2	0
2	0	1	0	3
3	0	1	2	0

We define an IVNS **A** in X as follows:

$$A_T = \begin{pmatrix} 0 & 1 & 2 & 3 \\ [0.9, 1] & [0.6, 0.8] & [0.5, 0.6] & [0.4, 0.6] \end{pmatrix},$$
$$A_I = \begin{pmatrix} 0 & 1 & 2 & 3 \\ [0, 0.1] & [0.1, 0.3] & [0.3, 0.4] & [0.5, 0.8] \end{pmatrix},$$
$$A_F = \begin{pmatrix} 0 & 1 & 2 & 3 \\ [0.8, 0.9] & [0.6, 0.8] & [0.5, 0.7] & [0.4, 0.6] \end{pmatrix}.$$

Then **A** is an interval-valued neutrosophic near UP-filter of X.

Definition 4.3.6 An IVNS **A** in X is called an *interval-valued neutrosophic UP-*

filter of X if it holds the following conditions: (4.3.4), (4.3.5), (4.3.6),

$$(\forall x, y \in X)(A_T(y) \succeq \text{rmin}\{A_T(x \cdot y), A_T(x)\}), \tag{4.3.10}$$

$$(\forall x, y \in X)(A_I(y) \preceq \text{rmax}\{A_I(x \cdot y), A_I(x)\}), \text{ and} \tag{4.3.11}$$

$$(\forall x, y \in X)(A_F(y) \succeq \text{rmin}\{A_F(x \cdot y), A_F(x)\}). \tag{4.3.12}$$

Example 4.3.7 Let $X = \{0, 1, 2, 3\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	3	3
2	0	1	0	0
3	0	1	2	0

We define an IVNS **A** in X as follows:

$$A_T = \begin{pmatrix} 0 & 1 & 2 & 3 \\ [0.9, 1] & [0.5, 0.8] & [0.3, 0.6] & [0.3, 0.6] \end{pmatrix},$$
$$A_I = \begin{pmatrix} 0 & 1 & 2 & 3 \\ [0, 0.1] & [0.2, 0.3] & [0.6, 0.8] & [0.6, 0.8] \end{pmatrix},$$
$$A_F = \begin{pmatrix} 0 & 1 & 2 & 3 \\ [0.8, 0.9] & [0.4, 0.5] & [0.3, 0.4] & [0.3, 0.4] \end{pmatrix}.$$

Then **A** is an interval-valued neutrosophic UP-filter of X.

Definition 4.3.8 An IVNS **A** in X is called an *interval-valued neutrosophic UP-ideal* of X if it holds the following conditions: (4.3.4), (4.3.5), (4.3.6),

$$(\forall x, y, z \in X)(A_T(x \cdot z) \succeq \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\}), \tag{4.3.13}$$

$$(\forall x, y, z \in X)(A_I(x \cdot z) \preceq \text{rmax}\{A_I(x \cdot (y \cdot z)), A_I(y)\}), \text{ and} \tag{4.3.14}$$

$$(\forall x, y, z \in X)(A_F(x \cdot z) \succeq \text{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\}). \tag{4.3.15}$$

Example 4.3.9 Let $X = \{0, 1, 2, 3\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	2	3
2	0	0	0	0
3	0	0	2	0

We define an IVNS $\mathbf{A}$ in X as follows:

$$A_T = \begin{pmatrix} 0 & 1 & 2 & 3 \\ [0.9, 1] & [0.7, 0.9] & [0.6, 0.8] & [0.6, 0.9] \end{pmatrix},$$
$$A_I = \begin{pmatrix} 0 & 1 & 2 & 3 \\ [0.1, 0.3] & [0.3, 0.5] & [0.4, 0.7] & [0.3, 0.6] \end{pmatrix},$$
$$A_F = \begin{pmatrix} 0 & 1 & 2 & 3 \\ [0.8, 0.9] & [0.5, 0.9] & [0.4, 0.6] & [0.5, 0.8] \end{pmatrix}.$$

Then $\mathbf{A}$ is an interval-valued neutrosophic UP-ideal of X.

Definition 4.3.10 An IVNS $\mathbf{A}$ in X is called an *interval-valued neutrosophic strong UP-ideal* of X if it holds the following conditions: (4.3.4), (4.3.5), (4.3.6),

$$(\forall x, y, z \in X)(A_T(x) \succeq \text{rmin}\{A_T((z \cdot y) \cdot (z \cdot x)), A_T(y)\}), \tag{4.3.16}$$

$$(\forall x, y, z \in X)(A_I(x) \preceq \text{rmax}\{A_I((z \cdot y) \cdot (z \cdot x)), A_I(y)\}), \tag{4.3.17}$$

$$(\forall x, y, z \in X)(A_F(x) \succeq \text{rmin}\{A_F((z \cdot y) \cdot (z \cdot x)), A_F(y)\}). \tag{4.3.18}$$

Example 4.3.11 Let $X = \{0, 1, 2, 3\}$ be a UP-algebra with a fixed element 0

and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	1	3
2	0	0	0	3
3	0	1	2	0

We define an IVNS $\mathbf{A}$ in X as follows:

$$(\forall x \in X)\begin{pmatrix} A_T(x) = [0.7, 0.9] \\ A_I(x) = [0.3, 0.5] \\ A_F(x) = [0.5, 0.9] \end{pmatrix}.$$

Then $\mathbf{A}$ is an interval-valued neutrosophic strong UP-ideal of X.

Definition 4.3.12 An IVNS $\mathbf{A}$ in a nonempty set X is said to be *constant* if $\mathbf{A}$ is a constant function from X to $[[0,1]]^3$. That is, A_T, A_I, and A_F are constant functions from X to $[[0,1]]$.

Theorem 4.3.13 *An IVNS* $\mathbf{A}$ *in* X *is constant if and only if it is an interval-valued neutrosophic strong UP-ideal of* X*.*

Proof. Assume that an IVNS $\mathbf{A}$ is constant in X. Then $A_T(x) = A_T(0), A_I(x) = A_I(0)$, and $A_F(x) = A_F(0)$ for all $x \in X$. Then for all $x \in X, A_T(0) \succeq A_T(x), A_T(0) \preceq A_I(x)$, and $A_F(0) \succeq A_F(x)$, and for all $x, y, z \in X$,

$$\begin{aligned} \text{rmin}\{A_T((z \cdot y) \cdot (z \cdot x)), A_T(y)\} &= \text{rmin}\{A_T(0), A_T(0)\} \\ &= A_T(0) \qquad ((2.0.15)) \\ &= A_T(x), \\ \text{rmax}\{A_I((z \cdot y) \cdot (z \cdot x)), A_I(y)\} &= \text{rmax}\{A_I(0), A_I(0)\} \end{aligned}$$

$$
\begin{aligned}
&= A_I(0) && ((2.0.15))\\
&= A_I(x),\\
\text{rmin}\{A_F((z \cdot y) \cdot (z \cdot x)), A_F(y)\} &= \text{rmin}\{A_F(0), A_F(0)\}\\
&= A_F(0) && ((2.0.15))\\
&= A_F(x).
\end{aligned}
$$

Hence, $\mathbf{A}$ is an interval-valued neutrosophic strong UP-ideal of X.

Conversely, assume that $\mathbf{A}$ is an interval-valued neutrosophic strong UP-ideal of X. Then for all $x \in X$,

$$
\begin{aligned}
A_T(x) &\succeq \text{rmin}\{A_T((x \cdot 0) \cdot (x \cdot x)), A_T(0)\}\\
&= \text{rmin}\{A_T(0 \cdot (x \cdot x)), A_T(0)\} && ((\text{UP-3}))\\
&= \text{rmin}\{A_T(x \cdot x), A_T(0)\} && ((\text{UP-2}))\\
&= \text{rmin}\{A_T(0), A_T(0)\} && ((3.0.1))\\
&= A_T(0) && ((2.0.15))\\
&\succeq A_T(x),\\
A_I(x) &\preceq \text{rmax}\{A_I((x \cdot 0) \cdot (x \cdot x)), A_I(0)\}\\
&= \text{rmax}\{A_I(0 \cdot (x \cdot x)), A_I(0)\} && ((\text{UP-3}))\\
&= \text{rmax}\{A_I(x \cdot x), A_I(0)\} && ((\text{UP-2}))\\
&= \text{rmax}\{A_I(0), A_I(0)\} && ((3.0.1))\\
&= A_I(0) && ((2.0.15))\\
&\preceq A_I(x),\\
A_F(x) &\succeq \text{rmin}\{A_F((x \cdot 0) \cdot (x \cdot x)), A_F(0)\}\\
&= \text{rmin}\{A_F(0 \cdot (x \cdot x)), A_F(0)\} && ((\text{UP-3}))\\
&= \text{rmin}\{A_F(x \cdot x), A_F(0)\} && ((\text{UP-2}))
\end{aligned}
$$

$$= \text{rmin}\{A_F(0), A_F(0)\} \qquad ((3.0.1))$$
$$= A_F(0) \qquad ((2.0.15))$$
$$\succeq A_F(x).$$

Thus $A_T(0) = A_T(x), A_I(0) = A_I(x)$, and $A_F(0) = A_F(x)$ for all $x \in X$. Hence, $\mathbf{A}$ is constant. □

Theorem 4.3.14 *Every interval-valued neutrosophic strong UP-ideal of X is an interval-valued neutrosophic UP-ideal.*

Proof. Assume that $\mathbf{A}$ is an interval-valued neutrosophic strong UP-ideal of X. Then for all $x \in X, A_T(0) \succeq A_T(x), A_T(0) \preceq A_I(x)$, and $A_F(0) \succeq A_F(x)$. Let $x, y, z \in X$. Then

$$A_T(x \cdot z) = A_T(y) \succeq \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\}, \qquad ((2.0.17))$$
$$A_I(x \cdot z) = A_I(y) \preceq \text{rmax}\{A_T(x \cdot (y \cdot z)), A_T(y)\}, \qquad ((2.0.17))$$
$$A_F(x \cdot z) = A_F(y) \succeq \text{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\}. \qquad ((2.0.17))$$

Hence, $\mathbf{A}$ is an interval-valued neutrosophic UP-ideal of X. □

The following example show that the converse of Theorem 4.3.14 is not true.

Example 4.3.15 From Example 4.3.9, we have $\mathbf{A}$ is an interval-valued neutrosophic UP-ideal of X. Since $A_T(1) = [0.7, 0.9] \not\succeq [0.9, 1] = \text{rmin}\{A_T((2 \cdot 0) \cdot (2 \cdot 1)), A_T(0)\}$, we have $\mathbf{A}$ is not an interval-valued neutrosophic strong UP-ideal of X.

Theorem 4.3.16 *Every interval-valued neutrosophic UP-ideal of X is an interval-valued neutrosophic UP-filter.*

Proof. Assume that **A** is an interval-valued neutrosophic UP-ideal of X. Then for all $x \in X, A_T(0) \succeq A_T(x), A_T(0) \preceq A_I(x)$, and $A_F(0) \succeq A_F(x)$. Let $x, y \in X$. Then

$$\begin{aligned} A_T(y) &= A_T(0 \cdot y) && \text{((UP-2))} \\ &\succeq \operatorname{rmin}\{A_T(0 \cdot (x \cdot y)), A_T(x)\} \\ &= \operatorname{rmin}\{A_T(x \cdot y), A_T(x)\}, && \text{((UP-2))} \\ A_I(y) &= A_I(0 \cdot y) && \text{((UP-2))} \\ &\preceq \operatorname{rmax}\{A_I(0 \cdot (x \cdot y)), A_I(x)\} \\ &= \operatorname{rmax}\{A_I(x \cdot y), A_I(x)\}, && \text{((UP-2))} \\ A_F(y) &= A_F(0 \cdot y) && \text{((UP-2))} \\ &\succeq \operatorname{rmin}\{A_F(0 \cdot (x \cdot y)), A_F(x)\} \\ &= \operatorname{rmin}\{A_F(x \cdot y), A_F(x)\}. && \text{((UP-2))} \end{aligned}$$

Hence, **A** is an interval-valued neutrosophic UP-filter of X. □

The following example show that the converse of Theorem 4.3.16 is not true.

Example 4.3.17 From Example 4.3.7, we have **A** is an interval-valued neutrosophic UP-filter of X. Since $A_I(3 \cdot 2) = [0.6, 0.8] \not\preceq [0.2, 0.3] = \operatorname{rmax}\{A_I(3 \cdot (1 \cdot 2)), A_I(1)\}$, we have **A** is not an interval-valued neutrosophic UP-ideal of X.

Theorem 4.3.18 *Every interval-valued neutrosophic UP-filter of X is an interval-valued neutrosophic near UP-filter.*

Proof. Assume that **A** is an interval-valued neutrosophic UP-filter of X. Then for all $x \in X, A_T(0) \succeq A_T(x), A_T(0) \preceq A_I(x)$, and $A_F(0) \succeq A_F(x)$. Let $x, y \in X$.

Then

$$
\begin{aligned}
A_T(x \cdot y) &\succeq \text{rmin}\{A_T(y \cdot (x \cdot y)), A_T(y)\} \\
&= \text{rmin}\{A_T(0), A_T(y)\} && ((3.0.5)) \\
&= A_T(y), \\
A_I(x \cdot y) &\preceq \text{rmax}\{A_I(y \cdot (x \cdot y)), A_I(y)\} \\
&= \text{rmax}\{A_I(0), A_I(y)\} && ((3.0.5)) \\
&= A_I(y), \\
A_F(x \cdot y) &\succeq \text{rmin}\{A_F(y \cdot (x \cdot y)), A_F(y)\} \\
&= \text{rmin}\{A_F(0), A_F(y)\} && ((3.0.5)) \\
&= A_F(y).
\end{aligned}
$$

Hence, **A** is an interval-valued neutrosophic near UP-filter of X. □

The following example show that the converse of Theorem 4.3.18 is not true.

Example 4.3.19 From Example 4.3.5, we have **A** is an interval-valued neutrosophic near UP-filter of X. Since $A_F(3) = [0.4, 0.6] \not\succeq [0.6, 0.8] = \text{rmin}\{A_F(1 \cdot 3), A_F(1)\}$, we have **A** is not an interval-valued neutrosophic UP-filter of X.

Theorem 4.3.20 *Every interval-valued neutrosophic near UP-filter of X is an interval-valued neutrosophic UP-subalgebra.*

Proof. Assume that **A** is an interval-valued neutrosophic near UP-filter of X. Then for all $x \in X, A_T(0) \succeq A_T(x), A_T(0) \preceq A_I(x)$, and $A_F(0) \succeq A_F(x)$. Let $x, y \in X$. By (2.0.17), we have

$$A_T(x \cdot y) \succeq A_T(y) \succeq \text{rmin}\{A_T(x), A_T(y)\},$$

$$A_I(x \cdot y) \preceq A_I(y) \preceq \text{rmax}\{A_I(x), A_I(y)\},$$

$$A_F(x \cdot y) \succeq A_F(y) \succeq \text{rmin}\{A_F(x), A_F(y)\}.$$

Hence, $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra of X. □

The following example show that the converse of Theorem 4.3.20 is not true.

Example 4.3.21 From Example 4.3.3, we have $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra of X. Since $A_F(1 \cdot 3) = [0.5, 0.7] \not\succeq [0.6, 0.8] = A_F(3)$, we have $\mathbf{A}$ is not an interval-valued neutrosophic near UP-filter of X.

Theorem 4.3.22 *If* $\mathbf{A}$ *is an interval-valued neutrosophic UP-subalgebra of* X *satisfying the following condition:*

$$(\forall x, y \in X)\left(x \cdot y \neq 0 \Rightarrow \begin{cases} A_T(x) \succeq A_T(y) \\ A_I(x) \preceq A_I(y) \\ A_F(x) \succeq A_F(y) \end{cases} \right), \tag{4.3.19}$$

then $\mathbf{A}$ *is an interval-valued neutrosophic near UP-filter of* X.

Proof. Assume that $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra of X satisfying the condition (4.3.19). By Theorem 4.3.2, we have $\mathbf{A}$ satisfies the conditions (4.3.4), (4.3.5), and (4.3.6). Next, let $x, y \in X$.

Case 1: $x \cdot y = 0$. Then

$$A_T(x \cdot y) = A_T(0) \succeq A_T(y), \qquad ((4.3.4))$$

$$A_I(x \cdot y) = A_I(0) \preceq A_I(y), \qquad ((4.3.5))$$

$$A_F(x \cdot y) = A_F(0) \succeq A_F(y). \qquad ((4.3.6))$$

Case 2: $x \cdot y \neq 0$. By (4.3.19), it follows that

$$\begin{aligned} A_T(x \cdot y) &\succeq \text{rmin}\{A_T(x), A_T(y)\} && ((4.3.1)) \\ &= A_T(y), && ((2.0.23)) \\ A_I(x \cdot y) &\preceq \text{rmax}\{A_I(x), A_I(y)\} && ((4.3.2)) \\ &= A_I(y), && ((2.0.24)) \\ A_F(x \cdot y) &\succeq \text{rmin}\{A_F(x), A_F(y)\} && ((4.3.3)) \\ &= A_F(y). && ((2.0.23)) \end{aligned}$$

Hence, **A** is an interval-valued neutrosophic near UP-filter of X. □

Theorem 4.3.23 *If* **A** *is an interval-valued neutrosophic near UP-filter of* X *satisfying the following condition:*

$$A_T = A_I = A_F, \tag{4.3.20}$$

then **A** *is an interval-valued neutrosophic strong UP-ideal of* X*.*

Proof. Assume that **A** is an interval-valued neutrosophic near UP-filter of X satisfying the condition (4.3.20). Then **A** satisfies the conditions (4.3.4), (4.3.5), and (4.3.6). Let $x \in X$. Then

$$\begin{aligned} A_T(0) \succeq A_T(x) = A_I(x) \succeq A_I(0) = A_T(0), \\ A_I(0) \preceq A_I(x) = A_T(x) \preceq A_T(0) = A_I(0), \\ A_F(0) \succeq A_F(x) = A_I(x) \succeq A_I(0) = A_F(0). \end{aligned}$$

Thus $A_T(0) = A_T(x), A_I(0) = A_I(x)$, and $A_F(0) = A_F(x)$, that is, **A** is constant. By Theorem 4.3.13, we have **A** is an interval-valued neutrosophic strong UP-ideal of X. □

Theorem 4.3.24 *If* $\mathbf{A}$ *is an interval-valued neutrosophic UP-filter of* X *satisfying the following condition:*

$$(\forall x, y, z \in X)\begin{pmatrix} A_T(y\cdot(x\cdot z)) = A_T(x\cdot(y\cdot z)) \\ A_I(y\cdot(x\cdot z)) = A_I(x\cdot(y\cdot z)) \\ A_F(y\cdot(x\cdot z)) = A_F(x\cdot(y\cdot z)) \end{pmatrix}, \tag{4.3.21}$$

then $\mathbf{A}$ *is an interval-valued neutrosophic UP-ideal of* X.

Proof. Assume that $\mathbf{A}$ is an interval-valued neutrosophic UP-filter of X satisfying the condition (4.3.21). Then $\mathbf{A}$ satisfies the conditions (4.3.4), (4.3.5), and (4.3.6). Next, let $x, y, z \in X$. Then

$$\begin{aligned} A_T(x\cdot z) &\succeq \operatorname{rmin}\{A_T(y\cdot(x\cdot z)), A_T(y)\} && ((4.3.10)) \\ &= \operatorname{rmin}\{A_T(x\cdot(y\cdot z)), A_T(y)\}, && ((4.3.21) \text{ for } A_T) \\ A_I(x\cdot z) &\preceq \operatorname{rmax}\{A_I(y\cdot(x\cdot z)), A_I(y)\} && ((4.3.11)) \\ &= \operatorname{rmax}\{A_I(x\cdot(y\cdot z)), A_I(y)\}, && ((4.3.21) \text{ for } A_I) \\ A_F(x\cdot z) &\succeq \operatorname{rmin}\{A_F(y\cdot(x\cdot z)), A_F(y)\} && ((4.3.12)) \\ &= \operatorname{rmin}\{A_F(x\cdot(y\cdot z)), A_F(y)\}. && ((4.3.21) \text{ for } A_F) \end{aligned}$$

Hence, $\mathbf{A}$ is an interval-valued neutrosophic UP-ideal of X. □

Theorem 4.3.25 *If* $\mathbf{A}$ *is an IVNS in* X *satisfying the following condition:*

$$(\forall x, y, z \in X)\left(z \leq x\cdot y \Rightarrow \begin{cases} A_T(z) \succeq \operatorname{rmin}\{A_T(x), A_T(y)\} \\ A_I(z) \preceq \operatorname{rmax}\{A_I(x), A_I(y)\} \\ A_F(z) \succeq \operatorname{rmin}\{A_F(x), A_F(y)\} \end{cases} \right), \tag{4.3.22}$$

then $\mathbf{A}$ *is an interval-valued neutrosophic UP-subalgebra of* X.

Proof. Assume that **A** is an IVNS in X satisfying the condition (4.3.22). Let $x, y \in X$. By (3.0.1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \leq x \cdot y$. It follows from (4.3.22) that

$$A_T(x \cdot y) \succeq \mathrm{rmin}\{A_T(x), A_T(y)\},$$
$$A_I(x \cdot y) \preceq \mathrm{rmax}\{A_I(x), A_I(y)\},$$
$$A_F(x \cdot y) \succeq \mathrm{rmin}\{A_F(x), A_F(y)\}.$$

Hence, **A** is an interval-valued neutrosophic UP-subalgebra of X. □

Theorem 4.3.26 *If* **A** *is an IVNS in* X *satisfying the following condition:*

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} A_T(y) \succeq \mathrm{rmin}\{A_T(z), A_T(x)\} \\ A_I(y) \preceq \mathrm{rmax}\{A_I(z), A_I(x)\} \\ A_F(y) \succeq \mathrm{rmin}\{A_F(z), A_F(x)\} \end{cases} \right), \tag{4.3.23}$$

then **A** *is an interval-valued neutrosophic UP-filter of* X.

Proof. Assume that **A** is an IVNS in X satisfying the condition (4.3.23). Let $x \in X$. By (UP-3), we have $x \cdot (x \cdot 0) = 0$, that is, $x \leq x \cdot 0$. It follows from (4.3.23) and (2.0.15) that

$$A_T(0) \succeq \mathrm{rmin}\{A_T(x), A_T(x)\} = A_T(x),$$
$$A_I(0) \preceq \mathrm{rmax}\{A_I(x), A_I(x)\} = A_I(x),$$
$$A_F(0) \succeq \mathrm{rmin}\{A_F(x), A_F(x)\} = A_F(x).$$

Next, let $x, y \in X$. By (3.0.1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \leq x \cdot y$. It follows from (4.3.23) that

$$A_T(y) \succeq \mathrm{rmin}\{A_T(x \cdot y), A_T(x)\},$$

$$A_I(y) \preceq \text{rmax}\{A_I(x \cdot y), A_I(x)\},$$

$$A_F(y) \succeq \text{rmin}\{A_F(x \cdot y), A_F(x)\}.$$

Hence, $\mathbf{A}$ is an interval-valued neutrosophic UP-filter of X. □

Theorem 4.3.27 *If* $\mathbf{A}$ *is an IVNS in* X *satisfying the following condition:*

$$(\forall a, x, y, z \in X) \left(a \leq x \cdot (y \cdot z) \Rightarrow \begin{cases} A_T(x \cdot z) \succeq \text{rmin}\{A_T(a), A_T(y)\} \\ A_I(x \cdot z) \preceq \text{rmax}\{A_I(a), A_I(y)\} \\ A_F(x \cdot z) \succeq \text{rmin}\{A_F(a), A_F(y)\} \end{cases} \right), \tag{4.3.24}$$

then $\mathbf{A}$ *is an interval-valued neutrosophic UP-ideal of* X*.*

Proof. Assume that $\mathbf{A}$ is an IVNS in X satisfying the condition (4.3.24). Let $x \in X$. By (UP-3), we have $x \cdot (0 \cdot (x \cdot 0)) = 0$, that is, $x \leq 0 \cdot (x \cdot 0)$. It follows from (4.3.24) and (2.0.15) that

$$A_T(0) = A_T(0 \cdot 0) \succeq \text{rmin}\{A_T(x), A_T(x)\} = A_T(x), \qquad \text{((UP-2))}$$

$$A_I(0) = A_I(0 \cdot 0) \preceq \text{rmax}\{A_I(x), A_I(x)\} = A_I(x), \qquad \text{((UP-2))}$$

$$A_F(0) = A_F(0 \cdot 0) \succeq \text{rmin}\{A_F(x), A_F(x)\} = A_F(x). \qquad \text{((UP-2))}$$

Next, let $x, y, z \in X$. By (3.0.1), we have $(x \cdot (y \cdot z)) \cdot (x \cdot (y \cdot z)) = 0$, that is, $x \cdot (y \cdot z) \leq x \cdot (y \cdot z)$. It follows from (4.3.24) that

$$A_T(x \cdot z) \succeq \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\},$$

$$A_I(x \cdot z) \preceq \text{rmax}\{A_I(x \cdot (y \cdot z)), A_I(y)\},$$

$$A_F(x \cdot z) \succeq \text{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\}.$$

Hence, $\mathbf{A}$ is an interval-valued neutrosophic UP-ideal of X. □

Theorem 4.3.28 *An IVNS A in X satisfies the following condition:*

$$(\forall x, y, z \in X) \left(z \leq x \cdot y \Rightarrow \begin{cases} A_T(z) \succeq A_T(y) \\ A_I(z) \preceq A_I(y) \\ A_F(z) \succeq A_F(y) \end{cases} \right) \tag{4.3.25}$$

if and only if **A** *is an interval-valued neutrosophic strong UP-ideal of X.*

Proof. Assume that **A** is an IVNS in X satisfying the condition (4.3.25). Let $x, y \in X$. By (UP-3) and (3.0.1), we have $x \cdot 0 = 0$, that is, $x \leq 0 = y \cdot y$. It follows from (4.3.25) that $A_T(x) \succeq A_T(y), A_I(x) \preceq A_I(y)$, and $A_F(x) \succeq A_F(y)$. Similarly, $A_T(y) \succeq A_T(x), A_I(y) \preceq A_I(x)$, and $A_F(y) \succeq A_F(x)$. Then $A_T(x) = A_T(y), A_I(x) = A_I(y)$, and $A_F(x) = A_F(y)$. Thus **A** is constant. By Theorem 4.3.13, we have **A** is an interval-valued neutrosophic strong UP-ideal of X.

The converse follows from Theorem 4.3.13. □

Then, we have the diagram of generalization of IVNSs in UP-algebras as shown in Figure 4.3.

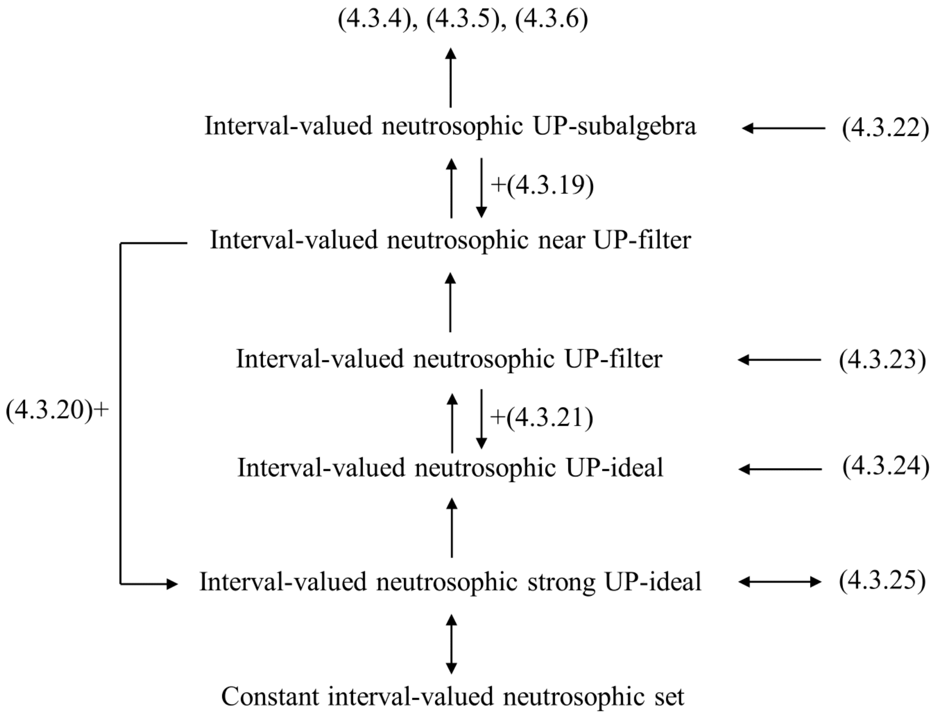

Figure 4.3: Interval-valued neutrosophic sets in UP-algebras

For any fixed interval numbers $\tilde{a}^+, \tilde{a}^-, \tilde{b}^+, \tilde{b}^-, \tilde{c}^+, \tilde{c}^- \in [[0,1]]$ such that $\tilde{a}^+ \succ \tilde{a}^-, \tilde{b}^+ \succ \tilde{b}^-, \tilde{c}^+ \succ \tilde{c}^-$ and a nonempty subset G of X, the IVNS $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ $= (X, A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}], A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}], A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}])$ in X, where $A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}], A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}]$, and $A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}]$ are IVFSs in X which are given as follows:

$$A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x) = \begin{cases} \tilde{a}^+ & \text{if } x \in G, \\ \tilde{a}^- & \text{otherwise,} \end{cases}$$

$$A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x) = \begin{cases} \tilde{b}^- & \text{if } x \in G, \\ \tilde{b}^+ & \text{otherwise,} \end{cases}$$

$$A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x) = \begin{cases} \tilde{c}^+ & \text{if } x \in G, \\ \tilde{c}^- & \text{otherwise.} \end{cases}$$

Lemma 4.3.29 *If the constant* 0 *of* X *is in a nonempty subset* G *of* X*, then the IVNS* $\mathbf{A}^{G}\left[\begin{smallmatrix}\tilde{a}^{+},\tilde{b}^{-},\tilde{c}^{+}\\ \tilde{a}^{-},\tilde{b}^{+},\tilde{c}^{-}\end{smallmatrix}\right]$ *in* X *satisfies the conditions* (4.3.4), (4.3.5), *and* (4.3.6).

Proof. If $0 \in G$, then $A_{T}^{G}\left[\begin{smallmatrix}\tilde{a}^{+}\\ \tilde{a}^{-}\end{smallmatrix}\right](0) = \tilde{a}^{+}, A_{I}^{G}\left[\begin{smallmatrix}\tilde{b}^{-}\\ \tilde{b}^{+}\end{smallmatrix}\right](0) = \tilde{b}^{-}$, and $A_{F}^{G}\left[\begin{smallmatrix}\tilde{c}^{+}\\ \tilde{c}^{-}\end{smallmatrix}\right](0) = \tilde{c}^{+}$. Thus

$$(\forall x \in X)\begin{pmatrix} A_{T}^{G}\left[\begin{smallmatrix}\tilde{a}^{+}\\ \tilde{a}^{-}\end{smallmatrix}\right](0) = \tilde{a}^{+} \succeq A_{T}^{G}\left[\begin{smallmatrix}\tilde{a}^{+}\\ \tilde{a}^{-}\end{smallmatrix}\right](x) \\ A_{I}^{G}\left[\begin{smallmatrix}\tilde{b}^{-}\\ \tilde{b}^{+}\end{smallmatrix}\right](0) = \tilde{b}^{-} \preceq A_{I}^{G}\left[\begin{smallmatrix}\tilde{b}^{-}\\ \tilde{b}^{+}\end{smallmatrix}\right](x) \\ A_{F}^{G}\left[\begin{smallmatrix}\tilde{c}^{+}\\ \tilde{c}^{-}\end{smallmatrix}\right](0) = \tilde{c}^{+} \succeq A_{F}^{G}\left[\begin{smallmatrix}\tilde{c}^{+}\\ \tilde{c}^{-}\end{smallmatrix}\right](x) \end{pmatrix}.$$

Hence, $\mathbf{A}^{G}\left[\begin{smallmatrix}\tilde{a}^{+},\tilde{b}^{-},\tilde{c}^{+}\\ \tilde{a}^{-},\tilde{b}^{+},\tilde{c}^{-}\end{smallmatrix}\right]$ satisfies the conditions (4.3.4), (4.3.5), and (4.3.6). □

Lemma 4.3.30 *If the IVNS* $\mathbf{A}^{G}\left[\begin{smallmatrix}\tilde{a}^{+},\tilde{b}^{-},\tilde{c}^{+}\\ \tilde{a}^{-},\tilde{b}^{+},\tilde{c}^{-}\end{smallmatrix}\right]$ *in* X *satisfies the condition* (4.3.4) *(resp.,* (4.3.5), (4.3.6)*), then the constant* 0 *of* X *is in* G.

Proof. Assume that the IVNS $\mathbf{A}^{G}\left[\begin{smallmatrix}\tilde{a}^{+},\tilde{b}^{-},\tilde{c}^{+}\\ \tilde{a}^{-},\tilde{b}^{+},\tilde{c}^{-}\end{smallmatrix}\right]$ in X satisfies the condition (4.3.4). Then $A_{T}^{G}\left[\begin{smallmatrix}\tilde{a}^{+}\\ \tilde{a}^{-}\end{smallmatrix}\right](0) \succeq A_{T}^{G}\left[\begin{smallmatrix}\tilde{a}^{+}\\ \tilde{a}^{-}\end{smallmatrix}\right](x)$ for all $x \in X$. Since G is nonempty, there exists $g \in G$. Thus $A_{T}^{G}\left[\begin{smallmatrix}\tilde{a}^{+}\\ \tilde{a}^{-}\end{smallmatrix}\right](g) = \tilde{a}^{+}$ and so $A_{T}^{G}\left[\begin{smallmatrix}\tilde{a}^{+}\\ \tilde{a}^{-}\end{smallmatrix}\right](0) \succeq A_{T}^{G}\left[\begin{smallmatrix}\tilde{a}^{+}\\ \tilde{a}^{-}\end{smallmatrix}\right](g) = \tilde{a}^{+} \succeq A_{T}^{G}\left[\begin{smallmatrix}\tilde{a}^{+}\\ \tilde{a}^{-}\end{smallmatrix}\right](0)$, that is, $A_{T}^{G}\left[\begin{smallmatrix}\tilde{a}^{+}\\ \tilde{a}^{-}\end{smallmatrix}\right](0) = \tilde{a}^{+}$. Hence, $0 \in G$. □

Theorem 4.3.31 *The IVNS* $\mathbf{A}^{G}\left[\begin{smallmatrix}\tilde{a}^{+},\tilde{b}^{-},\tilde{c}^{+}\\ \tilde{a}^{-},\tilde{b}^{+},\tilde{c}^{-}\end{smallmatrix}\right]$ *in* X *is an interval-valued neutrosophic UP-subalgebra of* X *if and only if a nonempty subset* G *of* X *is a UP-subalgebra of* X.

Proof. Assume that $\mathbf{A}^{G}\left[\begin{smallmatrix}\tilde{a}^{+},\tilde{b}^{-},\tilde{c}^{+}\\ \tilde{a}^{-},\tilde{b}^{+},\tilde{c}^{-}\end{smallmatrix}\right]$ is an interval-valued neutrosophic UP-subalgebra of X. Let $x, y \in G$. Then $A_{T}^{G}\left[\begin{smallmatrix}\tilde{a}^{+}\\ \tilde{a}^{-}\end{smallmatrix}\right](x) = \tilde{a}^{+} = A_{T}^{G}\left[\begin{smallmatrix}\tilde{a}^{+}\\ \tilde{a}^{-}\end{smallmatrix}\right](y)$. Thus

$$\begin{aligned} A_{T}^{G}\left[\begin{smallmatrix}\tilde{a}^{+}\\ \tilde{a}^{-}\end{smallmatrix}\right](x \cdot y) &\succeq \mathrm{rmin}\{A_{T}^{G}\left[\begin{smallmatrix}\tilde{a}^{+}\\ \tilde{a}^{-}\end{smallmatrix}\right](x), A_{T}^{G}\left[\begin{smallmatrix}\tilde{a}^{+}\\ \tilde{a}^{-}\end{smallmatrix}\right](y)\} && ((4.3.1)) \\ &= \mathrm{rmin}\{\tilde{a}^{+}, \tilde{a}^{+}\} \\ &= \tilde{a}^{+} && ((2.0.15)) \\ &\succeq A_{T}^{G}\left[\begin{smallmatrix}\tilde{a}^{+}\\ \tilde{a}^{-}\end{smallmatrix}\right](x \cdot y) \end{aligned}$$

and so $A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot y) = \tilde{a}^+$. Thus $x \cdot y \in G$. Hence, G is a UP-subalgebra of X.

Conversely, assume that G is a UP-subalgebra of X. Let $x, y \in X$.

Case 1: $x, y \in G$. Then

$$A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x) = \tilde{a}^+ = A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](y),$$
$$A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x) = \tilde{b}^- = A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](y),$$
$$A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x) = \tilde{c}^+ = A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](y).$$

Since G is a UP-subalgebra of X, we have $x \cdot y \in G$ and so $A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot y) = \tilde{a}^+$, $A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x \cdot y) = \tilde{b}^-$, and $A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x \cdot y) = \tilde{c}^+$. By (2.0.15), it follows that

$$A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot y) = \tilde{a}^+ \succeq \tilde{a}^+ = \text{rmin}\{\tilde{a}^+, \tilde{a}^+\} = \text{rmin}\{A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x), A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](y)\},$$
$$A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x \cdot y) = \tilde{b}^- \preceq \tilde{b}^- = \text{rmax}\{\tilde{b}^-, \tilde{b}^-\} = \text{rmax}\{A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x), A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](y)\},$$
$$A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x \cdot y) = \tilde{c}^+ \succeq \tilde{c}^+ = \text{rmin}\{\tilde{c}^+, \tilde{c}^+\} = \text{rmin}\{A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x), A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](y)\}.$$

Case 2: $x \notin G$ or $y \notin G$. Then

$$A_T^G[^{\tilde{a}^-}_{\tilde{a}^-}](x) = \tilde{a}^- \text{ or } A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](y) = \tilde{a}^-,$$
$$A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x) = \tilde{b}^+ \text{ or } A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](y) = \tilde{b}^+,$$
$$A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x) = \tilde{c}^- \text{ or } A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](y) = \tilde{c}^-.$$

By (2.0.15), it follows that

$$\text{rmin}\{A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x), A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](y)\} = \tilde{a}^-,$$
$$\text{rmax}\{A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x), A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](y)\} = \tilde{b}^+,$$
$$\text{rmin}\{A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x), A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](y)\} = \tilde{c}^-.$$

Therefore,

$$A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x\cdot y) \succeq \tilde{a}^- = \text{rmin}\{A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x), A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](y)\},$$
$$A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x\cdot y) \preceq \tilde{b}^+ = \text{rmax}\{A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x), A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](y)\},$$
$$A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x\cdot y) \succeq \tilde{c}^- = \text{rmin}\{A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x), A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](y)\}.$$

Hence, $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ is an interval-valued neutrosophic UP-subalgebra of X. □

Theorem 4.3.32 *The IVNS* $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ *in* X *is an interval-valued neutrosophic near UP-filter of* X *if and only if a nonempty subset* G *of* X *is a near UP-filter of* X*.*

Proof. Assume that $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ is an interval-valued neutrosophic near UP-filter of X. Since $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ satisfies the condition (4.3.4), it follows from Lemma 4.3.30 that $0 \in G$. Next, let $x \in X$ and $y \in G$. Then $A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](y) = \tilde{a}^+$. By (4.3.7)

$$A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x\cdot y) \succeq A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](y) = \tilde{a}^+ \succeq A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x\cdot y)$$

and so $A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x\cdot y) = \tilde{a}^+$. Thus $x\cdot y \in G$. Hence, G is a near UP-filter of X.

Conversely, assume that G is a near UP-filter of X. Since $0 \in G$, it follows from Lemma 4.3.29 that $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ satisfies the conditions (4.3.4), (4.3.5), and (4.3.6). Next, let $x, y \in X$.

Case 1: $y \in G$. Then $A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](y) = \tilde{a}^+, A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](y) = \tilde{b}^-$, and $A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](y) = \tilde{c}^+$. Since G is a near UP-filter of X, we have $x\cdot y \in G$ and so $A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x\cdot y) = \tilde{a}^+, A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x\cdot y) = \tilde{b}^-$, and $A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x\cdot y) = \tilde{c}^+$. Thus

$$A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x\cdot y) = \tilde{a}^+ \succeq \tilde{a}^+ = A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](y),$$
$$A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x\cdot y) = \tilde{b}^- \preceq \tilde{b}^- = A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](y),$$
$$A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x\cdot y) = \tilde{c}^+ \succeq \tilde{c}^+ = A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](y).$$

Case 2: $y \notin G$. Then $A_T^G[{}^{\tilde{a}^+}_{\tilde{a}^-}](y) = \tilde{a}^-$, $A_I^G[{}^{\tilde{b}^-}_{\tilde{b}^+}](y) = \tilde{b}^+$, and $A_F^G[{}^{\tilde{c}^+}_{\tilde{c}^-}](y) = \tilde{c}^-$. Thus

$$A_T^G[{}^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot y) \succeq \tilde{a}^- = A_T^G[{}^{\tilde{a}^+}_{\tilde{a}^-}](y),$$
$$A_I^G[{}^{\tilde{b}^-}_{\tilde{b}^+}](x \cdot y) \preceq \tilde{b}^+ = A_I^G[{}^{\tilde{b}^-}_{\tilde{b}^+}](y),$$
$$A_F^G[{}^{\tilde{c}^+}_{\tilde{c}^-}](x \cdot y) \succeq \tilde{c}^- = A_F^G[{}^{\tilde{c}^+}_{\tilde{c}^-}](y).$$

Hence, $\mathbf{A}^G[{}^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ is an interval-valued neutrosophic near UP-filter of X. □

Theorem 4.3.33 *The IVNS $\mathbf{A}^G[{}^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ in X is an interval-valued neutrosophic UP-filter of X if and only if a nonempty subset G of X is a UP-filter of X.*

Proof. Assume that $\mathbf{A}^G[{}^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ is an interval-valued neutrosophic UP-filter of X. Since $\mathbf{A}^G[{}^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ satisfies the condition (4.3.4), it follows from Lemma 4.3.30 that $0 \in G$. Next, let $x, y \in X$ be such that $x \cdot y \in G$ and $x \in G$. Then $A_T^G[{}^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot y) = \tilde{a}^+ = A_T^G[{}^{\tilde{a}^+}_{\tilde{a}^-}](x)$. Thus

$$\begin{aligned} A_T^G[{}^{\tilde{a}^+}_{\tilde{a}^-}](y) &\succeq \text{rmin}\{A_T^G[{}^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot y), A_T^G[{}^{\tilde{a}^+}_{\tilde{a}^-}](x)\} && ((4.3.10)) \\ &= \text{rmin}\{\tilde{a}^+, \tilde{a}^+\} \\ &= \tilde{a}^+ && ((2.0.15)) \\ &\succeq A_T^G[{}^{\tilde{a}^+}_{\tilde{a}^-}](y) \end{aligned}$$

and so $A_T^G[{}^{\tilde{a}^+}_{\tilde{a}^-}](y) = \tilde{a}^+$. Thus $y \in G$. Hence, G is a UP-filter of X.

Conversely, assume that G is a UP-filter of X. Since $0 \in G$, it follows from Lemma 4.3.29 that $\mathbf{A}^G[{}^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ satisfies the conditions (4.3.4), (4.3.5), and (4.3.6). Next, let $x, y \in X$.

Case 1: $x \cdot y \in G$ and $x \in G$. Then

$$A_T^G[\tfrac{\tilde{a}^+}{\tilde{a}^-}](x \cdot y) = \tilde{a}^+ = A_T^G[\tfrac{\tilde{a}^+}{\tilde{a}^-}](x),$$
$$A_I^G[\tfrac{\tilde{b}^-}{\tilde{b}^+}](x \cdot y) = \tilde{b}^- = A_I^G[\tfrac{\tilde{b}^-}{\tilde{b}^+}](x),$$
$$A_F^G[\tfrac{\tilde{c}^+}{\tilde{c}^-}](x \cdot y) = \tilde{c}^+ = A_F^G[\tfrac{\tilde{c}^+}{\tilde{c}^-}](x).$$

Since G is a UP-filter of X, we have $y \in G$ and so $A_T^G[\tfrac{\tilde{a}^+}{\tilde{a}^-}](y) = \tilde{a}^+$, $A_I^G[\tfrac{\tilde{b}^-}{\tilde{b}^+}](y) = \tilde{b}^-$, and $A_F^G[\tfrac{\tilde{c}^+}{\tilde{c}^-}](y) = \tilde{c}^+$. By (2.0.15), it follows that

$$A_T^G[\tfrac{\tilde{a}^+}{\tilde{a}^-}](y) = \tilde{a}^+ \succeq \tilde{a}^+ = \text{rmin}\{\tilde{a}^+, \tilde{a}^+\} = \text{rmin}\{A_T^G[\tfrac{\tilde{a}^+}{\tilde{a}^-}](x \cdot y), A_T^G[\tfrac{\tilde{a}^+}{\tilde{a}^-}](x)\},$$
$$A_I^G[\tfrac{\tilde{b}^-}{\tilde{b}^+}](y) = \tilde{b}^- \preceq \tilde{b}^- = \text{rmax}\{\tilde{b}^-, \tilde{b}^-\} = \text{rmax}\{A_I^G[\tfrac{\tilde{b}^-}{\tilde{b}^+}](x \cdot y), A_I^G[\tfrac{\tilde{b}^-}{\tilde{b}^+}](x)\},$$
$$A_F^G[\tfrac{\tilde{c}^+}{\tilde{c}^-}](y) = \tilde{c}^+ \succeq \tilde{c}^+ = \text{rmin}\{\tilde{c}^+, \tilde{c}^+\} = \text{rmin}\{A_F^G[\tfrac{\tilde{c}^+}{\tilde{c}^-}](x \cdot y), A_F^G[\tfrac{\tilde{c}^+}{\tilde{c}^-}](x)\}.$$

Case 2: $x \cdot y \notin G$ or $x \notin G$. Then

$$A_T^G[\tfrac{\tilde{a}^+}{\tilde{a}^-}](x \cdot y) = \tilde{a}^- \text{ or } A_T^G[\tfrac{\tilde{a}^+}{\tilde{a}^-}](x) = \tilde{a}^-,$$
$$A_I^G[\tfrac{\tilde{b}^-}{\tilde{b}^+}](x \cdot y) = \tilde{b}^+ \text{ or } A_I^G[\tfrac{\tilde{b}^-}{\tilde{b}^+}](x) = \tilde{b}^+,$$
$$A_F^G[\tfrac{\tilde{c}^+}{\tilde{c}^-}](x \cdot y) = \tilde{c}^- \text{ or } A_F^G[\tfrac{\tilde{c}^+}{\tilde{c}^-}](x) = \tilde{c}^-.$$

By (2.0.15), it follows that

$$\text{rmin}\{A_T^G[\tfrac{\tilde{a}^+}{\tilde{a}^-}](x \cdot y), A_T^G[\tfrac{\tilde{a}^+}{\tilde{a}^-}](x)\} = \tilde{a}^-,$$
$$\text{rmax}\{A_I^G[\tfrac{\tilde{b}^-}{\tilde{b}^+}](x \cdot y), A_I^G[\tfrac{\tilde{b}^-}{\tilde{b}^+}](x)\} = \tilde{b}^+,$$
$$\text{rmin}\{A_F^G[\tfrac{\tilde{c}^+}{\tilde{c}^-}](x \cdot y), A_F^G[\tfrac{\tilde{c}^+}{\tilde{c}^-}](x)\} = \tilde{c}^-.$$

Therefore,

$$A_T^G[\tfrac{\tilde{a}^+}{\tilde{a}^-}](y) \succeq \tilde{a}^- = \text{rmin}\{A_T^G[\tfrac{\tilde{a}^+}{\tilde{a}^-}](x \cdot y), A_T^G[\tfrac{\tilde{a}^+}{\tilde{a}^-}](x)\},$$

$$A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](y) \preceq \tilde{b}^+ = \text{rmax}\{A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x \cdot y), A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x)\},$$

$$A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](y) \succeq \tilde{c}^- = \text{rmin}\{A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x \cdot y), A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x)\}.$$

Hence, $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ is an interval-valued neutrosophic UP-filter of X. □

Theorem 4.3.34 *The IVNS $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ in X is an interval-valued neutrosophic UP-ideal of X if and only if a nonempty subset G of X is a UP-ideal of X.*

Proof. Assume that $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ is an interval-valued neutrosophic UP-ideal of X. Since $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ satisfies the condition (4.3.4), it follows from Lemma 4.3.30 that $0 \in G$. Next, let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in G$ and $y \in G$. Then $A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot (y \cdot z)) = \tilde{a}^+ = A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](y)$. Thus

$$\begin{aligned} A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot z) &\succeq \text{rmin}\{A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot (y \cdot z)), A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](y)\} && ((4.3.13)) \\ &= \text{rmin}\{\tilde{a}^+, \tilde{a}^+\} \\ &= \tilde{a}^+ && ((2.0.15)) \\ &\succeq A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot z) \end{aligned}$$

and so $A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot z) = \tilde{a}^+$. Thus $x \cdot z \in G$. Hence, G is a UP-ideal of X.

Conversely, assume that G is a UP-ideal of X. Since $0 \in G$, it follows from Lemma 4.3.29 that $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ satisfies the conditions (4.3.4), (4.3.5), and (4.3.6). Next, let $x, y, z \in X$.

Case 1: $x \cdot (y \cdot z) \in G$ and $y \in G$. Then

$$A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot (y \cdot z)) = \tilde{a}^+ = A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](y),$$

$$A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x \cdot (y \cdot z)) = \tilde{b}^- = A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](y),$$

$$A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x \cdot (y \cdot z)) = \tilde{c}^+ = A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](y).$$

Since G is a UP-ideal of X, we have $x \cdot z \in G$ and so $A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot z) = \tilde{a}^+$, $A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x \cdot z) = \tilde{b}^-$, and $A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x \cdot z) = \tilde{c}^+$. By (2.0.15), it follows that

$$A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot z) = \tilde{a}^+ \succeq \tilde{a}^+ = \text{rmin}\{\tilde{a}^+, \tilde{a}^+\} = \text{rmin}\{A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot (y \cdot z)), A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](y)\},$$
$$A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x \cdot z) = \tilde{b}^- \preceq \tilde{b}^- = \text{rmax}\{\tilde{b}^-, \tilde{b}^-\} = \text{rmax}\{A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x \cdot (y \cdot z)), A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](y)\},$$
$$A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x \cdot z) = \tilde{c}^+ \succeq \tilde{c}^+ = \text{rmin}\{\tilde{c}^+, \tilde{c}^+\} = \text{rmin}\{A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x \cdot (y \cdot z)), A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](y)\}.$$

Case 2: $x \cdot (y \cdot z) \notin G$ or $y \notin G$. Then

$$A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot (y \cdot z)) = \tilde{a}^- \text{ or } A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](y) = \tilde{a}^-,$$
$$A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x \cdot (y \cdot z)) = \tilde{b}^+ \text{ or } A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](y) = \tilde{b}^+,$$
$$A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x \cdot (y \cdot z)) = \tilde{c}^- \text{ or } A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](y) = \tilde{c}^-.$$

By (2.0.15), it follows that

$$\text{rmin}\{A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot (y \cdot z)), A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](y)\} = \tilde{a}^-,$$
$$\text{rmax}\{A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x \cdot (y \cdot z)), A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](y)\} = \tilde{b}^+,$$
$$\text{rmin}\{A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x \cdot (y \cdot z)), A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](y)\} = \tilde{c}^-.$$

Therefore,

$$A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot z) \succeq \tilde{a}^- = \text{rmin}\{A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot (y \cdot z)), A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](y)\},$$
$$A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x \cdot z) \preceq \tilde{b}^+ = \text{rmax}\{A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x \cdot (y \cdot z)), A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](y)\},$$
$$A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x \cdot z) \succeq \tilde{c}^- = \text{rmin}\{A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x \cdot (y \cdot z)), A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](y)\}.$$

Hence, $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ is an interval-valued neutrosophic UP-ideal of X. □

Theorem 4.3.35 *The IVNS* $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ *in* X *is an interval-valued neutrosophic strong UP-ideal of* X *if and only if a nonempty subset* G *of* X *is a strong UP-ideal* *of* X*.*

Proof. Assume that $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ is an interval-valued neutrosophic strong UP-ideal of X. By Theorem 4.3.13, we have $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ is constant, that is, $A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}]$ is constant. Since G is nonempty, we have $A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](x) = \tilde{a}^+$ for all $x \in X$. Thus $G = X$. Hence, G is a strong UP-ideal of X.

Conversely, assume that G is a strong UP-ideal of X. Then $G = X$, so

$$(\forall x \in X)\begin{pmatrix} A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](x) = \tilde{a}^+ \\ A^G_I[^{\tilde{b}^-}_{\tilde{b}^+}](x) = \tilde{b}^- \\ A^G_F[^{\tilde{c}^+}_{\tilde{c}^-}](x) = \tilde{c}^+ \end{pmatrix}.$$

Thus $A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}]$, $A^G_I[^{\tilde{b}^-}_{\tilde{b}^+}]$, and $A^G_F[^{\tilde{c}^+}_{\tilde{c}^-}]$ are constant, that is, $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ is constant. By Theorem 4.3.13, we have $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ is an interval-valued neutrosophic strong UP-ideal of X. □

In the next order, we also discuss the relationships among interval-valued neutrosophic UP-subalgebras (resp., interval-valued neutrosophic near UP-filters, interval-valued neutrosophic UP-filters, interval-valued neutrosophic UP-ideals, interval-valued neutrosophic strong UP-ideals) of UP-algebras and their level subsets.

Definition 4.3.36 Let A be an IVFS in a nonempty set X. For any $\tilde{a} \in [[0,1]]$, the sets

$$U(A;\tilde{a}) = \{x \in X \mid A(x) \succeq \tilde{a}\},$$

$$L(A;\tilde{a}) = \{x \in X \mid A(x) \preceq \tilde{a}\},$$

$$E(A;\tilde{a}) = \{x \in X \mid A(x) = \tilde{a}\}$$

are called an upper $\tilde{a}$-level subset, a lower $\tilde{a}$-level subset, and an equal $\tilde{a}$-level subset of A, respectively.

Theorem 4.3.37 *An IVNS* $\mathbf{A}$ *in* X *is an interval-valued neutrosophic UP-subalgebra of* X *if and only if for all* $\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$*, the sets* $U(A_T; \tilde{a}), L(A_I; \tilde{b})$*, and* $U(A_F; \tilde{c})$ *are either empty or UP-subalgebras of* X*.*

Proof. Assume that $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra of X. Let $\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$ be such that $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are nonempty.

Let $x, y \in U(A_T; \tilde{a})$. Then $A_T(x) \succeq \tilde{a}$ and $A_T(y) \succeq \tilde{a}$. Since $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra of X and by (2.0.20), we have

$$A_T(x \cdot y) \succeq \text{rmin}\{A_T(x), A_T(y)\} \succeq \tilde{a}.$$

Thus $x \cdot y \in U(A_T; \tilde{a})$.

Let $x, y \in L(A_I; \tilde{b})$. Then $A_I(x) \preceq \tilde{b}$ and $A_I(y) \preceq \tilde{b}$. Since $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra of X and by (2.0.22), we have

$$A_I(x \cdot y) \preceq \text{rmax}\{A_I(x), A_I(y)\} \preceq \tilde{b}.$$

Thus $x \cdot y \in L(A_I; \tilde{b})$.

Let $x, y \in U(A_F; \tilde{c})$. Then $A_F(x) \succeq \tilde{c}$ and $A_F(y) \succeq \tilde{c}$. Since $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra of X and by (2.0.20), we have

$$A_F(x \cdot y) \succeq \text{rmin}\{A_F(x), A_F(y)\} \succeq \tilde{c}.$$

Thus $x \cdot y \in U(A_F; \tilde{c})$.

Hence, $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are UP-subalgebras of X.

Conversely, assume that for all $\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$, the sets $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are either empty or UP-subalgebras of X.

Let $x, y \in X$. By (2.0.17), we have $A_T(x) \succeq \text{rmin}\{A_T(x), A_T(y)\}$ and $A_T(y) \succeq \text{rmin}\{A_T(x), A_T(y)\}$. Thus $x, y \in U(A_T; \text{rmin}\{A_T(x), A_T(y)\})$. By assumption, we have $U(A_T; \text{rmin}\{A_T(x), A_T(y)\})$ is a UP-subalgebra of X. Then $x \cdot y \in U(A_T; \text{rmin}\{A_T(x), A_T(y)\})$. Thus $A_T(x \cdot y) \succeq \text{rmin}\{A_T(x), A_T(y)\}$.

Let $x, y \in X$. By (2.0.17), we have $A_I(x) \preceq \text{rmax}\{A_I(x), A_I(y)\}$ and $A_I(y) \preceq \text{rmax}\{A_I(x), A_I(y)\}$. Thus $x, y \in L(A_I; \text{rmax}\{A_I(x), A_I(y)\})$. By assumption, we have $L(A_I; \text{rmax}\{A_I(x), A_I(y)\})$ is a UP-subalgebra of X. Then $x \cdot y \in L(A_I; \text{rmax}\{A_I(x), A_I(y)\})$. Thus $A_I(x \cdot y) \preceq \text{rmax}\{A_I(x), A_I(y)\}$.

Let $x, y \in X$. By (2.0.17), we have $A_F(x) \succeq \text{rmin}\{A_F(x), A_F(y)\}$ and $A_F(y) \succeq \text{rmin}\{A_F(x), A_F(y)\}$. Thus $x, y \in U(A_F; \text{rmin}\{A_F(x), A_F(y)\})$. By assumption, we have $U(A_F; \text{rmin}\{A_F(x), A_F(y)\})$ is a UP-subalgebra of X. Then $x \cdot y \in U(A_F; \text{rmin}\{A_F(x), A_F(y)\})$. Thus $A_F(x \cdot y) \succeq \text{rmin}\{A_F(x), A_F(y)\}$.

Hence, $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra of X. □

Theorem 4.3.38 *An IVNS* $\mathbf{A}$ *in* X *is an interval-valued neutrosophic near UP-filter of* X *if and only if for all* $\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$, *the sets* $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, *and* $U(A_F; \tilde{c})$ *are either empty or near UP-filters of* X.

Proof. Assume that $\mathbf{A}$ is an interval-valued neutrosophic near UP-filter of X. Let $\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$ be such that $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are nonempty.

Let $x \in U(A_T; \tilde{a}), y \in L(A_I; \tilde{b}), z \in U(A_F; \tilde{c})$. Since $\mathbf{A}$ is an interval-valued neutrosophic near UP-filter of X, we have

$$A_T(0) \succeq A_T(x) \succeq \tilde{a},\ A_I(0) \preceq A_I(y) \preceq \tilde{b},\ A_F(0) \succeq A_F(z) \succeq \tilde{c}.$$

Thus $0 \in U(A_T; \tilde{a}), 0 \in L(A_I; \tilde{b})$, and $0 \in U(A_T; \tilde{a})$.

Let $x \in X$ and $y \in U(A_T; \tilde{a})$. Then $A_T(y) \succeq \tilde{a}$. Since $\mathbf{A}$ is an interval-valued neutrosophic near UP-filter of X, we have

$$A_T(x \cdot y) \succeq A_T(y) \succeq \tilde{a}.$$

Thus $x \cdot y \in U(A_T; \tilde{a})$.

Let $x \in X$ and $y \in L(A_I; \tilde{b})$. Then $A_I(y) \preceq \tilde{b}$. Since $\mathbf{A}$ is an interval-valued neutrosophic near UP-filter of X, we have

$$A_I(x \cdot y) \preceq A_I(y) \preceq \tilde{b}.$$

Thus $x \cdot y \in L(A_I; \tilde{b})$.

Let $x \in X$ and $y \in U(A_F; \tilde{c})$. Then $A_F(y) \succeq \tilde{c}$. Since $\mathbf{A}$ is an interval-valued neutrosophic near UP-filter of X, we have

$$A_F(x \cdot y) \succeq A_F(y) \succeq \tilde{c}.$$

Thus $x \cdot y \in U(A_F; \tilde{c})$.

Hence, $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are near UP-filters of X.

Conversely, assume that for all $\tilde{a}, \tilde{b}, \tilde{c} \in [[0, 1]]$, the sets $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are either empty or near UP-filters of X.

Let $x \in X$. Then $x \in U(A_T; A_T(x)) \neq \emptyset, x \in L(A_I; A_I(x)) \neq \emptyset$, and $x \in U(A_T; A_T(x)) \neq \emptyset$. By assumption, we have $U(A_T; A_T(x)), L(A_I; A_I(x))$, and $U(A_F; A_F(x))$ are near UP-filters of X. Then $0 \in U(A_T; A_T(x)), 0 \in L(A_I; A_I(x))$, and $0 \in U(A_F; A_F(x))$. Thus $A_T(0) \succeq A_T(x), A_I(0) \preceq A_I(x)$, and $A_F(0) \succeq$

$A_F(x)$.

Let $x, y \in X$. Then $y \in U(A_T; A_T(y)) \neq \emptyset$. By assumption, we have $U(A_T; A_T(y))$ is a near UP-filter of X. Then $x \cdot y \in U(A_T; A_T(y))$. Thus $A_T(x \cdot y) \succeq A_T(y)$.

Let $x, y \in X$. Then $y \in L(A_I; A_I(y)) \neq \emptyset$. By assumption, we have $L(A_I; A_I(y))$ is a near UP-filter of X. Then $x \cdot y \in L(A_I; A_I(y))$. Thus $A_I(x \cdot y) \preceq A_I(y)$.

Let $x, y \in X$. Then $y \in U(A_F; A_F(y)) \neq \emptyset$. By assumption, we have $U(A_F; A_F(y))$ is a near UP-filter of X. Then $x \cdot y \in U(A_F; A_F(y))$. Thus $A_F(x \cdot y) \succeq A_F(y)$.

Hence, **A** is an interval-valued neutrosophic near UP-filter of X. □

Theorem 4.3.39 *An IVNS* **A** *in X is an interval-valued neutrosophic UP-filter of X if and only if for all $\tilde{a}, \tilde{b}, \tilde{c} \in [[0, 1]]$, the sets $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are either empty or UP-filters of X.*

Proof. Assume that **A** is an interval-valued neutrosophic UP-filter of X. Let $\tilde{a}$, $\tilde{b}, \tilde{c} \in [[0, 1]]$ be such that $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are nonempty.

Let $x \in U(A_T; \tilde{a}), y \in L(A_I; \tilde{b}), z \in U(A_F; \tilde{c})$. Since **A** is an interval-valued neutrosophic UP-filter of X, we have

$$A_T(0) \succeq A_T(x) \succeq \tilde{a},\ \ A_I(0) \preceq A_I(y) \preceq \tilde{b},\ \ A_F(0) \succeq A_F(z) \succeq \tilde{c}.$$

Thus $0 \in U(A_T; \tilde{a}), 0 \in L(A_I; \tilde{b})$, and $0 \in U(A_T; \tilde{a})$.

Let $x, y \in X$ be such that $x \cdot y, x \in U(A_T; \tilde{a})$. Then $A_T(x \cdot y) \succeq \tilde{a}$ and

$A_T(x) \succeq \tilde{a}$. Since $\mathbf{A}$ is an interval-valued neutrosophic UP-filter of X, we have

$$A_T(y) \succeq \text{rmin}\{A_T(x \cdot y), A_T(x)\} \succeq \tilde{a}.$$

Thus $y \in U(A_T; \tilde{a})$.

Let $x, y \in X$ be such that $x \cdot y, x \in L(A_I; \tilde{b})$. Then $A_I(x \cdot y) \preceq \tilde{b}$ and $A_I(x) \preceq \tilde{b}$. Since $\mathbf{A}$ is an interval-valued neutrosophic UP-filter of X, we have

$$A_I(y) \preceq \text{rmax}\{A_I(x \cdot y), A_I(x)\} \preceq \tilde{b}.$$

Thus $y \in L(A_I; \tilde{b})$.

Let $x, y \in X$ be such that $x \cdot y, x \in U(A_F; \tilde{c})$. Then $A_F(x \cdot y) \succeq \tilde{c}$ and $A_F(x) \succeq \tilde{c}$. Since $\mathbf{A}$ is an interval-valued neutrosophic UP-filter of X, we have

$$A_F(y) \succeq \text{rmin}\{A_F(x \cdot y), A_F(x)\} \succeq \tilde{c}.$$

Thus $y \in U(A_F; \tilde{c})$.

Hence, $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are UP-filters of X.

Conversely, assume that for all $\tilde{a}, \tilde{b}, \tilde{c} \in [[0, 1]]$, the sets $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are either empty or UP-filters of X.

Let $x \in X$. Then $x \in U(A_T; A_T(x)) \neq \emptyset, x \in L(A_I; A_I(x)) \neq \emptyset$, and $x \in U(A_T; A_T(x)) \neq \emptyset$. By assumption, we have $U(A_T; A_T(x)), L(A_I; A_I(x))$, and $U(A_F; A_F(x))$ are UP-filters of X. Then $0 \in U(A_T; A_T(x)), 0 \in L(A_I; A_I(x))$, and $0 \in U(A_F; A_F(x))$. Thus $A_T(0) \succeq A_T(x), A_I(0) \preceq A_I(x)$, and $A_F(0) \succeq A_F(x)$.

Let $x, y \in X$. By (2.0.17), we have $A_T(x \cdot y) \succeq \text{rmin}\{A_T(x \cdot y), A_T(x)\}$ and $A_T(x) \succeq \text{rmin}\{A_T(x \cdot y), A_T(x)\}$. Thus $x \cdot y, x \in U(A_T; \text{rmin}\{A_T(x \cdot y), A_T(x)\})$.

By assumption, we have $U(A_T; \text{rmin}\{A_T(x \cdot y), A_T(x)\})$ is a UP-filter of X. Then $y \in U(A_T; \text{rmin}\{A_T(x \cdot y), A_T(x)\})$. Thus $A_T(y) \succeq \text{rmin}\{A_T(x \cdot y), A_T(x)\}$.

Let $x, y \in X$. By (2.0.17), we have $A_I(x \cdot y) \preceq \text{rmax}\{A_I(x \cdot y), A_I(x)\}$ and $A_I(x) \preceq \text{rmax}\{A_I(x \cdot y), A_I(x)\}$. Thus $x \cdot y, x \in L(A_I; \text{rmax}\{A_I(x \cdot y), A_I(x)\})$. By assumption, we have $L(A_I; \text{rmax}\{A_I(x \cdot y), A_I(x)\})$ is a UP-filter of X. Then $y \in L(A_I; \text{rmax}\{A_I(x \cdot y), A_I(x)\})$. Thus $A_I(y) \preceq \text{rmax}\{A_I(x \cdot y), A_I(x)\}$.

Let $x, y \in X$. By (2.0.17), we have $A_F(x \cdot y) \succeq \text{rmin}\{A_F(x \cdot y), A_F(x)\}$ and $A_F(x) \succeq \text{rmin}\{A_F(x \cdot y), A_F(x)\}$. Thus $x \cdot y, x \in U(A_F; \text{rmin}\{A_F(x \cdot y), A_F(x)\})$. By assumption, we have $U(A_F; \text{rmin}\{A_F(x \cdot y), A_F(x)\})$ is a UP-filter of X. Then $y \in U(A_F; \text{rmin}\{A_F(x \cdot y), A_F(x)\})$. Thus $A_F(y) \succeq \text{rmin}\{A_F(x \cdot y), A_F(x)\}$.

Hence, $\mathbf{A}$ is an interval-valued neutrosophic UP-filter of X. □

Theorem 4.3.40 *An IVNS* $\mathbf{A}$ *in* X *is an interval-valued neutrosophic UP-ideal of* X *if and only if for all* $\tilde{a}, \tilde{b}, \tilde{c} \in [[0, 1]]$, *the sets* $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, *and* $U(A_F; \tilde{c})$ *are either empty or UP-ideals of* X.

Proof. Assume that $\mathbf{A}$ is an interval-valued neutrosophic UP-ideal of X. Let $\tilde{a}, \tilde{b}, \tilde{c} \in [[0, 1]]$ be such that $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are nonempty.

Let $x \in U(A_T; \tilde{a}), y \in L(A_I; \tilde{b}), z \in U(A_F; \tilde{c})$. Since $\mathbf{A}$ is an interval-valued neutrosophic UP-ideal of X, we have

$$A_T(0) \succeq A_T(x) \succeq \tilde{a}, \; A_I(0) \preceq A_I(y) \preceq \tilde{b}, \; A_F(0) \succeq A_F(z) \succeq \tilde{c}.$$

Thus $0 \in U(A_T; \tilde{a}), 0 \in L(A_I; \tilde{b})$, and $0 \in U(A_T; \tilde{a})$.

Let $x, y, z \in X$ be such that $x \cdot (y \cdot z), y \in U(A_T; \tilde{a})$. Then $A_T(x \cdot (y \cdot z)) \succeq \tilde{a}$ and $A_T(y) \succeq \tilde{a}$. Since $\mathbf{A}$ is an interval-valued neutrosophic UP-ideal of X, we have

$$A_T(x \cdot z) \succeq \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\} \succeq \tilde{a}.$$

Thus $x \cdot z \in U(A_T; \tilde{a})$.

Let $x, y, z \in X$ be such that $x \cdot (y \cdot z), y \in L(A_I; \tilde{b})$. Then $A_I(x \cdot (y \cdot z)) \preceq \tilde{b}$ and $A_I(y) \preceq \tilde{b}$. Since **A** is an interval-valued neutrosophic UP-ideal of X, we have

$$A_I(x \cdot z) \preceq \text{rmax}\{A_I(x \cdot (y \cdot z)), A_I(y)\} \preceq \tilde{b}.$$

Thus $x \cdot z \in L(A_I; \tilde{b})$.

Let $x, y, z \in X$ be such that $x \cdot (y \cdot z), y \in U(A_F; \tilde{c})$. Then $A_F(x \cdot (y \cdot z)) \succeq \tilde{c}$ and $A_F(y) \succeq \tilde{c}$. Since **A** is an interval-valued neutrosophic UP-ideal of X, we have

$$A_F(x \cdot z) \succeq \text{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\} \succeq \tilde{c}.$$

Thus $x \cdot z \in U(A_F; \tilde{c})$.

Hence, $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are UP-ideals of X.

Conversely, assume that for all $\tilde{a}, \tilde{b}, \tilde{c} \in [[0, 1]]$, the sets $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are either empty or UP-ideals of X.

Let $x \in X$. Then $x \in U(A_T; A_T(x)) \neq \emptyset, x \in L(A_I; A_I(x)) \neq \emptyset$, and $x \in U(A_T; A_T(x)) \neq \emptyset$. By assumption, we have $U(A_T; A_T(x)), L(A_I; A_I(x))$, and $U(A_F; A_F(x))$ are UP-ideals of X. Then $0 \in U(A_T; A_T(x)), 0 \in L(A_I; A_I(x))$, and $0 \in U(A_F; A_F(x))$. Thus $A_T(0) \succeq A_T(x), A_I(0) \preceq A_I(x)$, and $A_F(0) \succeq A_F(x)$.

Let $x, y \in X$. By (2.0.17), we have $A_T(x \cdot (y \cdot z)) \succeq \text{rmin}\{A_T(x \cdot (y \cdot$

$z)), A_T(y)\}$ and $A_T(y) \succeq \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\}$. Thus $x \cdot (y \cdot z), y \in U(A_T; \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\})$. By assumption, we have $U(A_T; \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\})$ is a UP-ideal of X. Then $x \cdot z \in U(A_T; \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\})$. Thus $A_T(x \cdot z) \succeq \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\}$.

Let $x, y \in X$. By (2.0.17), we have $A_I(x \cdot (y \cdot z)) \preceq \text{rmax}\{A_I(x \cdot (y \cdot z)), A_I(y)\}$ and $A_I(y) \preceq \text{rmax}\{A_I(x \cdot (y \cdot z)), A_I(y)\}$. Thus $x \cdot (y \cdot z), y \in L(A_I; \text{rmax}\{A_I(x \cdot (y \cdot z)), A_I(y)\})$. By assumption, we have $L(A_I; \text{rmax}\{A_I(x \cdot (y \cdot z)), A_I(x)\})$ is a UP-ideal of X. Then $x \cdot z \in L(A_I; \text{rmax}\{A_I(x \cdot (y \cdot z)), A_I(y)\})$. Thus $A_I(x \cdot z) \preceq \text{rmax}\{A_I(x \cdot (y \cdot z)), A_I(y)\}$.

Let $x, y \in X$. By (2.0.17), we have $A_F(x \cdot (y \cdot z)) \succeq \text{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\}$ and $A_F(y) \succeq \text{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\}$. Thus $x \cdot (y \cdot z), y \in U(A_F; \text{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\})$. By assumption, we have $U(A_F; \text{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\})$ is a UP-ideal of X. Then $x \cdot z \in U(A_F; \text{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\})$. Thus $A_F(x \cdot z) \succeq \text{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\}$.

Hence, $\mathbf{A}$ is an interval-valued neutrosophic UP-ideal of X. □

Theorem 4.3.41 *An IVNS $\mathbf{A}$ in X is an interval-valued neutrosophic strong UP-ideal if and only if for all $\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$, the sets $E(A_T; A_T(0)), E(A_I; A_I(0))$, and $E(A_F; A_F(0))$ are strong UP-ideals of X.*

Proof. Assume that $\mathbf{A}$ is an interval-valued neutrosophic strong UP-ideal of X. By Theorem 4.3.13, we have $\mathbf{A}$ is constant, that is, A_T, A_I, A_F are constant. Thus

$$(\forall x \in X)\begin{pmatrix} A_T(x) = A_T(0) \\ A_I(x) = A_I(0) \\ A_F(x) = A_F(0) \end{pmatrix}.$$

Hence, $E(A_T; A_T(0)) = X, E(A_I; A_I(0)) = X$, and $E(A_F; A_F(0)) = X$ and so $E(A_T; A_T(0)), E(A_I; A_I(0))$, and $E(A_F; A_F(0))$ are strong UP-ideals of X.

Conversely, assume that $E(A_T; A_T(0)), E(A_I; A_I(0))$, and $E(A_F; A_F(0))$ are strong UP-ideals of X. Then $E(A_T; A_T(0)) = X, E(A_I; A_I(0)) = X$, and $E(A_F; A_F(0)) = X$ and so

$$(\forall x \in X)\begin{pmatrix} A_T(x) = A_T(0) \\ A_I(x) = A_I(0) \\ A_F(x) = A_F(0) \end{pmatrix}.$$

Thus A_T, A_I, A_F are constant, that is, $\mathbf{A}$ is constant. By Theorem 4.3.13, we have $\mathbf{A}$ is an interval-valued neutrosophic strong UP-ideal of X. □

4.4 Neutrosophic cubic sets in UP-algebras

In this section, we introduce the mixed concepts of neutrosophic cubic UP-subalgebras, neutrosophic cubic near UP-filters, neutrosophic cubic UP-filters, neutrosophic cubic UP-ideals, and neutrosophic cubic strong UP-ideals of UP-algebras, provide the necessary examples, investigate their properties, and prove their generalizations.

Definition 4.4.1 A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X is called a *neutrosophic cubic UP-subalgebra* of X if it holds the following conditions:

$$(\forall x, y \in X)\begin{pmatrix} A_T(x \cdot y) \succeq \text{rmin}\{A_T(x), A_T(y)\} \\ A_I(x \cdot y) \preceq \text{rmax}\{A_I(x), A_I(y)\} \\ A_F(x \cdot y) \succeq \text{rmin}\{A_F(x), A_F(y)\} \end{pmatrix} \tag{4.4.1}$$

and

$$(\forall x, y \in X)\begin{pmatrix} \lambda_T(x \cdot y) \leq \max\{\lambda_T(x), \lambda_T(y)\} \\ \lambda_I(x \cdot y) \geq \min\{\lambda_I(x), \lambda_I(y)\} \\ \lambda_F(x \cdot y) \leq \max\{\lambda_F(x), \lambda_F(y)\} \end{pmatrix}. \tag{4.4.2}$$

Proposition 4.4.2 *If $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-subalgebra of X, then*

$$(\forall x \in X)\begin{pmatrix} A_T(0) \succeq A_T(x) \\ A_I(0) \preceq A_I(x) \\ A_F(0) \succeq A_F(x) \end{pmatrix} \tag{4.4.3}$$

and

$$(\forall x \in X)\begin{pmatrix} \lambda_T(0) \leq \lambda_T(x) \\ \lambda_I(0) \geq \lambda_I(x) \\ \lambda_F(0) \leq \lambda_F(x) \end{pmatrix}. \tag{4.4.4}$$

Proof. Let $\mathscr{A} = (\mathbf{A}, \Lambda)$ be a neutrosophic cubic UP-subalgebra of X. By (3.0.1) and (2.0.15), we have

$$(\forall x \in X)\begin{pmatrix} A_T(0) = A_T(x \cdot x) \succeq \mathrm{rmin}\{A_T(x), A_T(x)\} = A_T(x) \\ A_I(0) = A_I(x \cdot x) \preceq \mathrm{rmax}\{A_I(x), A_I(x)\} = A_I(x) \\ A_F(0) = A_F(x \cdot x) \succeq \mathrm{rmin}\{A_F(x), A_F(x)\} = A_F(x) \\ \lambda_T(0) = \lambda_T(x \cdot x) \leq \max\{\lambda_T(x), \lambda_T(x)\} = \lambda_T(x) \\ \lambda_I(0) = \lambda_I(x \cdot x) \geq \min\{\lambda_I(x), \lambda_I(x)\} = \lambda_I(x) \\ \lambda_F(0) = \lambda_F(x \cdot x) \leq \max\{\lambda_F(x), \lambda_F(x)\} = \lambda_F(x) \end{pmatrix}.$$

□

Example 4.4.3 Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0

and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	1	3	4
2	0	0	0	3	4
3	0	0	0	0	4
4	0	0	0	0	0

We define a NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X with the tabular representation as follows:

X	$\mathbf{A}(x)$	$\Lambda(x)$
0	$([1,1],[0,0.3],[0.7,1])$	$(0,1,0)$
1	$([0.6,0.7],[0.4,0.5],[0.4,0.5])$	$(0.3,0.2,0.4)$
2	$([0.4,0.8],[0.1,0.4],[0.5,0.7])$	$(0.5,0.6,0.2)$
3	$([0.3,0.4],[0.8,0.9],[0.2,0.3])$	$(0.7,0.8,0.7)$
4	$([0.7,0.8],[0.2,0.4],[0.6,0.7])$	$(0.5,0.4,0.8)$

Then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-subalgebra of X.

Definition 4.4.4 A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X is called a *neutrosophic cubic near UP-filter* of X if it holds the following conditions: (4.4.3), (4.4.4),

$$(\forall x, y \in X) \begin{pmatrix} A_T(x \cdot y) \succeq A_T(y) \\ A_I(x \cdot y) \preceq A_I(y) \\ A_F(x \cdot y) \succeq A_F(y) \end{pmatrix} \tag{4.4.5}$$

and

$$(\forall x, y \in X) \begin{pmatrix} \lambda_T(x \cdot y) \leq \lambda_T(y) \\ \lambda_I(x \cdot y) \geq \lambda_I(y) \\ \lambda_F(x \cdot y) \leq \lambda_F(y) \end{pmatrix}. \tag{4.4.6}$$

Example 4.4.5 Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	1	2	4
2	0	0	0	1	4
3	0	0	0	0	4
4	0	1	2	3	0

We define a NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X with the tabular representation as follows:

X	$\mathbf{A}(x)$	$\Lambda(x)$
0	$([0.9, 1], [0, 0.1], [1, 1])$	$(0, 0.9, 0.1)$
1	$([0.6, 0.8], [0.1, 0.3], [0.6, 0.8])$	$(0.3, 0.8, 0.2)$
2	$([0.5, 0.6], [0.3, 0.4], [0.5, 0.7])$	$(0.5, 0.7, 0.6)$
3	$([0.4, 0.6], [0.5, 0.6], [0.4, 0.6])$	$(0.6, 0.3, 0.7)$
4	$([0.1, 0.7], [0.8, 0.9], [0.1, 0.3])$	$(0.2, 0.4, 0.5)$

Then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic near UP-filter of X.

Definition 4.4.6 A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X is called a *neutrosophic cubic UP-filter* of X if it holds the following conditions: (4.4.3), (4.4.4),

$$(\forall x, y \in X) \begin{pmatrix} A_T(y) \succeq \text{rmin}\{A_T(x \cdot y), A_T(x)\} \\ A_I(y) \preceq \text{rmax}\{A_I(x \cdot y), A_I(x)\} \\ A_F(y) \succeq \text{rmin}\{A_F(x \cdot y), A_F(x)\} \end{pmatrix} \tag{4.4.7}$$

and

$$(\forall x, y \in X)\begin{pmatrix} \lambda_T(y) \leq \max\{\lambda_T(x \cdot y), \lambda_T(x)\} \\ \lambda_I(y) \geq \min\{\lambda_I(x \cdot y), \lambda_I(x)\} \\ \lambda_F(y) \leq \max\{\lambda_F(x \cdot y), \lambda_F(x)\} \end{pmatrix}. \tag{4.4.8}$$

Example 4.4.7 Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	0	0	3	3
3	0	1	2	0	3
4	0	1	2	0	0

We define a NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X with the tabular representation as follows:

X	$\mathbf{A}(x)$	$\Lambda(x)$
0	$([0.9, 1], [0, 0.1], [0.8, 0.9])$	$(0, 1, 0.1)$
1	$([0.5, 0.8], [0.2, 0.3], [0.6, 0.7])$	$(0.2, 0.7, 0.2)$
2	$([0.3, 0.7], [0.4, 0.5], [0.5, 0.6])$	$(0.5, 0.5, 0.9)$
3	$([0.1, 0.4], [0.7, 0.9], [0.2, 0.4])$	$(0.7, 0.4, 0.3)$
4	$([0.1, 0.4], [0.7, 0.9], [0.2, 0.4])$	$(0.7, 0.4, 0.3)$

Then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-filter of X.

Definition 4.4.8 A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X is called a *neutrosophic cubic UP-*

ideal of X if it holds the following conditions: (4.4.3), (4.4.4),

$$(\forall x, y, z \in X)\begin{pmatrix} A_T(x \cdot z) \succeq \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\} \\ A_I(x \cdot z) \preceq \text{rmax}\{A_I(x \cdot (y \cdot z)), A_I(y)\} \\ A_F(x \cdot z) \succeq \text{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\} \end{pmatrix} \tag{4.4.9}$$

and

$$(\forall x, y, z \in X)\begin{pmatrix} \lambda_T(x \cdot z) \leq \max\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\} \\ \lambda_I(x \cdot z) \geq \min\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\} \\ \lambda_F(x \cdot z) \leq \max\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\} \end{pmatrix}. \tag{4.4.10}$$

Example 4.4.9 Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	0	0	0	4
3	0	0	2	0	4
4	0	0	0	0	0

We define a NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X with the tabular representation as follows:

X	$\mathbf{A}(x)$	$\Lambda(x)$
0	$([0.9, 1], [0.1, 0.3], [0.8, 0.9])$	$(0, 1, 0)$
1	$([0.7, 0.9], [0.3, 0.5], [0.5, 0.9])$	$(0.3, 0.6, 0.2)$
2	$([0.6, 0.8], [0.4, 0.7], [0.4, 0.6])$	$(0.5, 0.5, 0.7)$
3	$([0.6, 0.9], [0.3, 0.6], [0.5, 0.8])$	$(0.4, 0.6, 0.4)$
4	$([0.3, 0.5], [0.5, 0.9], [0.4, 0.5])$	$(0.6, 0.2, 0.9)$

Then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-ideal of X.

Definition 4.4.10 A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X is called a *neutrosophic cubic strong UP-ideal* of X if it holds the following conditions: (4.4.3), (4.4.4),

$$(\forall x, y, z \in X) \begin{pmatrix} A_T(x) \succeq \text{rmin}\{A_T((z \cdot y) \cdot (z \cdot x)), A_T(y)\} \\ A_I(x) \preceq \text{rmax}\{A_I((z \cdot y) \cdot (z \cdot x)), A_I(y)\} \\ A_F(x) \succeq \text{rmin}\{A_F((z \cdot y) \cdot (z \cdot x)), A_F(y)\} \end{pmatrix} \tag{4.4.11}$$

and

$$(\forall x, y, z \in X) \begin{pmatrix} \lambda_T(x) \leq \max\{\lambda_T((z \cdot y) \cdot (z \cdot x)), \lambda_T(y)\} \\ \lambda_I(x) \geq \min\{\lambda_I((z \cdot y) \cdot (z \cdot x)), \lambda_I(y)\} \\ \lambda_F(x) \leq \max\{\lambda_F((z \cdot y) \cdot (z \cdot x)), \lambda_F(y)\} \end{pmatrix}. \tag{4.4.12}$$

Example 4.4.11 Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	1	0	3	4
3	0	1	0	0	4
4	0	1	0	3	0

We define a NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X with the tabular representation as follows:

X	$\mathbf{A}(x)$	$\Lambda(x)$
0	$([0.5, 0.7], [0.3, 0.9], [0.4, 0.5])$	$(0.5, 0.4, 0.7)$
1	$([0.5, 0.7], [0.3, 0.9], [0.4, 0.5])$	$(0.5, 0.4, 0.7)$
2	$([0.5, 0.7], [0.3, 0.9], [0.4, 0.5])$	$(0.5, 0.4, 0.7)$
3	$([0.5, 0.7], [0.3, 0.9], [0.4, 0.5])$	$(0.5, 0.4, 0.7)$
4	$([0.5, 0.7], [0.3, 0.9], [0.4, 0.5])$	$(0.5, 0.4, 0.7)$

Then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic strong UP-ideal of X.

Theorem 4.4.12 *A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X is a neutrosophic cubic UP-subalgebra (resp., neutrosophic cubic near UP-filter, neutrosophic cubic UP-filter, neutrosophic cubic UP-ideal, neutrosophic cubic strong UP-ideal) of X if and only if the IVNS $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra (resp., interval-valued neutrosophic near UP-filter, interval-valued neutrosophic UP-filter, interval-valued neutrosophic UP-ideal, interval-valued neutrosophic strong UP-ideal) of X and the NS Λ is a special neutrosophic UP-subalgebra (resp., special neutrosophic near UP-filter, special neutrosophic UP-filter, special neutrosophic UP-ideal, special neutrosophic strong UP-ideal) of X.*

Proof. It is straightforward by Definitions 4.1.1 and 4.2.1. □

Theorem 4.4.13 *A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X is constant if and only if it is a neutrosophic cubic strong UP-ideal of X.*

Proof. Assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a constant neutrosophic cubic set in X. Then $A_T(x) = A_T(0), A_I(x) = A_I(0), A_F(x) = A_F(0), \lambda_T(x) = \lambda_T(0), \lambda_I(x) = \lambda_I(0)$, and $\lambda_F(x) = \lambda_F(0)$ for all $x \in X$. Then for all $x \in X, A_T(0) \succeq A_T(x), A_I(0) \preceq A_I(x), A_F(0) \succeq A_F(x), \lambda_T(0) \leq \lambda_T(x), \lambda_I(0) \geq \lambda_I(x)$, and $\lambda_F(0) \leq \lambda_F(x)$, and for all $x, y, z \in X$,

$$\begin{aligned}
\text{rmin}\{A_T((z \cdot y) \cdot (z \cdot x)), A_T(y)\} &= \text{rmin}\{A_T(0), A_T(0)\} \\
&= A_T(0) && ((2.0.15)) \\
&= A_T(x), \\
\text{rmax}\{A_I((z \cdot y) \cdot (z \cdot x)), A_I(y)\} &= \text{rmax}\{A_I(0), A_I(0)\} \\
&= A_I(0) && ((2.0.15)) \\
&= A_I(x), \\
\text{rmin}\{A_F((z \cdot y) \cdot (z \cdot x)), A_F(y)\} &= \text{rmin}\{A_F(0), A_F(0)\} \\
&= A_F(0) && ((2.0.15)) \\
&= A_F(x), \\
\max\{\lambda_T((z \cdot y) \cdot (z \cdot x)), \lambda_T(y)\} &= \max\{\lambda_T(0), \lambda_T(0)\} \\
&= \lambda_T(0) \\
&= \lambda_T(x), \\
\min\{\lambda_I((z \cdot y) \cdot (z \cdot x)), \lambda_I(y)\} &= \min\{\lambda_I(0), \lambda_I(0)\} \\
&= \lambda_I(0) \\
&= \lambda_I(x), \\
\max\{\lambda_F((z \cdot y) \cdot (z \cdot x)), \lambda_F(y)\} &= \max\{\lambda_F(0), \lambda_F(0)\} \\
&= \lambda_F(0) \\
&= \lambda_F(x).
\end{aligned}$$

Hence, $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic strong UP-ideal of X.

Conversely, assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic strong UP-ideal of X. Then for all $x \in X$,

$$A_T(x) \succeq \text{rmin}\{A_T((x \cdot 0) \cdot (x \cdot x)), A_T(0)\}$$

$$
\begin{aligned}
&= \operatorname{rmin}\{A_T(0 \cdot (x \cdot x)), A_T(0)\} && \text{((UP-3))} \\
&= \operatorname{rmin}\{A_T(x \cdot x), A_T(0)\} && \text{((UP-2))} \\
&= \operatorname{rmin}\{A_T(0), A_T(0)\} && \text{((3.0.1))} \\
&= A_T(0) && \text{((2.0.15))} \\
&\succeq A_T(x), \\
A_I(x) &\preceq \operatorname{rmax}\{A_I((x \cdot 0) \cdot (x \cdot x)), A_I(0)\} \\
&= \operatorname{rmax}\{A_I(0 \cdot (x \cdot x)), A_I(0)\} && \text{((UP-3))} \\
&= \operatorname{rmax}\{A_I(x \cdot x), A_I(0)\} && \text{((UP-2))} \\
&= \operatorname{rmax}\{A_I(0), A_I(0)\} && \text{((3.0.1))} \\
&= A_I(0) && \text{((2.0.15))} \\
&\preceq A_I(x), \\
A_F(x) &\succeq \operatorname{rmin}\{A_F((x \cdot 0) \cdot (x \cdot x)), A_F(0)\} \\
&= \operatorname{rmin}\{A_F(0 \cdot (x \cdot x)), A_F(0)\} && \text{((UP-3))} \\
&= \operatorname{rmin}\{A_F(x \cdot x), A_F(0)\} && \text{((UP-2))} \\
&= \operatorname{rmin}\{A_F(0), A_F(0)\} && \text{((3.0.1))} \\
&= A_F(0) && \text{((2.0.15))} \\
&\succeq A_F(x), \\
\lambda_T(x) &\leq \max\{\lambda_T((x \cdot 0) \cdot (x \cdot x)), \lambda_T(0)\} \\
&= \max\{\lambda_T(0 \cdot (x \cdot x)), \lambda_T(0)\} && \text{((UP-3))} \\
&= \max\{\lambda_T(x \cdot x), \lambda_T(0)\} && \text{((UP-2))} \\
&= \max\{\lambda_T(0), \lambda_T(0)\} && \text{((3.0.1))} \\
&= \lambda_T(0) \\
&\leq \lambda_T(x), \\
\lambda_I(x) &\geq \min\{\lambda_I((x \cdot 0) \cdot (x \cdot x)), \lambda_I(0)\} \\
&= \min\{\lambda_I(0 \cdot (x \cdot x)), \lambda_I(0)\} && \text{((UP-3))}
\end{aligned}
$$

$$= \min\{\lambda_I(x \cdot x), \lambda_I(0)\} \quad \text{((UP-2))}$$
$$= \min\{\lambda_I(0), \lambda_I(0)\} \quad \text{((3.0.1))}$$
$$= \lambda_I(0)$$
$$\geq \lambda_I(x),$$
$$\lambda_F(x) \leq \max\{\lambda_F((x \cdot 0) \cdot (x \cdot x)), \lambda_F(0)\}$$
$$= \max\{\lambda_F(0 \cdot (x \cdot x)), \lambda_F(0)\} \quad \text{((UP-3))}$$
$$= \max\{\lambda_F(x \cdot x), \lambda_F(0)\} \quad \text{((UP-2))}$$
$$= \max\{\lambda_F(0), \lambda_F(0)\} \quad \text{((3.0.1))}$$
$$= \lambda_F(0)$$
$$\leq \lambda_F(x).$$

Thus $A_T(0) = A_T(x), A_I(0) = A_I(x), A_F(0) = A_F(x), \lambda_T(0) = \lambda_T(x), \lambda_I(0) = \lambda_I(x)$, and $\lambda_F(0) = \lambda_F(x)$ for all $x \in X$. Hence, $\mathscr{A} = (\mathbf{A}, \Lambda)$ is constant. □

Theorem 4.4.14 *Every neutrosophic cubic strong UP-ideal of X is a neutrosophic cubic UP-ideal.*

Proof. Assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic strong UP-ideal of X. Then for all $x \in X, A_T(0) \succeq A_T(x), A_I(0) \preceq A_I(x), A_F(0) \succeq A_F(x), \lambda_T(0) \leq \lambda_T(x), \lambda_I(0) \geq \lambda_I(x)$, and $\lambda_F(0) \leq \lambda_F(x)$. Let $x, y, z \in X$. Then

$$A_T(x \cdot z) = A_T(y) \succeq \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\}, \quad \text{((2.0.17))}$$
$$A_I(x \cdot z) = A_I(y) \preceq \text{rmax}\{A_T(x \cdot (y \cdot z)), A_T(y)\}, \quad \text{((2.0.17))}$$
$$A_F(x \cdot z) = A_F(y) \succeq \text{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\}, \quad \text{((2.0.17))}$$
$$\lambda_T(x \cdot z) = A_T(y) \leq \max\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\},$$
$$\lambda_I(x \cdot z) = A_I(y) \geq \min\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\},$$
$$\lambda_F(x \cdot z) = A_F(y) \leq \max\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}.$$

Hence, $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-ideal of X. $\square$

The following example show that the converse of Theorem 4.4.14 is not true.

Example 4.4.15 From Example 4.4.9, we have $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-ideal of X. Since $\lambda_F(3) = 0.6 > 0.3 = \max\{\lambda_F((2 \cdot 0) \cdot (2 \cdot 3)), \lambda_F(0)\}$, we have $\mathscr{A} = (\mathbf{A}, \Lambda)$ is not a neutrosophic cubic strong UP-ideal of X.

Theorem 4.4.16 *Every neutrosophic cubic UP-ideal of X is a neutrosophic cubic UP-filter.*

Proof. Assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-ideal of X. Then for all $x \in X, A_T(0) \succeq A_T(x), A_I(0) \preceq A_I(x), A_F(0) \succeq A_F(x), \lambda_T(0) \leq \lambda_T(x), \lambda_I(0) \geq \lambda_I(x)$, and $\lambda_F(0) \leq \lambda_F(x)$. Let $x, y \in X$. Then

$$A_T(y) = A_T(0 \cdot y) \quad \text{((UP-2))}$$
$$\succeq \text{rmin}\{A_T(0 \cdot (x \cdot y)), A_T(x)\}$$
$$= \text{rmin}\{A_T(x \cdot y), A_T(x)\}, \quad \text{((UP-2))}$$
$$A_I(y) = A_I(0 \cdot y) \quad \text{((UP-2))}$$
$$\preceq \text{rmax}\{A_I(0 \cdot (x \cdot y)), A_I(x)\}$$
$$= \text{rmax}\{A_I(x \cdot y), A_I(x)\}, \quad \text{((UP-2))}$$
$$A_F(y) = A_F(0 \cdot y) \quad \text{((UP-2))}$$
$$\succeq \text{rmin}\{A_F(0 \cdot (x \cdot y)), A_F(x)\}$$
$$= \text{rmin}\{A_F(x \cdot y), A_F(x)\}, \quad \text{((UP-2))}$$
$$\lambda_T(y) = \lambda_T(0 \cdot y) \quad \text{((UP-2))}$$
$$\leq \max\{\lambda_T(0 \cdot (x \cdot y)), \lambda_T(x)\}$$
$$= \max\{\lambda_T(x \cdot y), \lambda_T(x)\}, \quad \text{((UP-2))}$$
$$\lambda_I(y) = \lambda_I(0 \cdot y) \quad \text{((UP-2))}$$

$$\geq \min\{\lambda_I(0\cdot(x\cdot y)), \lambda_I(x)\}$$

$$= \max\{\lambda_I(x\cdot y), \lambda_I(x)\}, \quad ((\text{UP-2}))$$

$$\lambda_F(y) = \lambda_F(0\cdot y) \quad ((\text{UP-2}))$$

$$\leq \max\{\lambda_F(0\cdot(x\cdot y)), \lambda_F(x)\}$$

$$= \max\{\lambda_F(x\cdot y), \lambda_F(x)\}. \quad ((\text{UP-2}))$$

Hence, $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-filter of X. □

The following example show that the converse of Theorem 4.4.16 is not true.

Example 4.4.17 From Example 4.4.7, we have $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-filter of X. Since $A_F(3\cdot 4) = [0.2, 0.4] \not\succeq [0.5, 0.6] = \text{rmin}\{A_F(3\cdot(2\cdot 4)), A_F(2)\}$, we have $\mathscr{A} = (\mathbf{A}, \Lambda)$ is not a neutrosophic cubic UP-ideal of X.

Theorem 4.4.18 *Every neutrosophic cubic UP-filter of X is a neutrosophic cubic near UP-filter.*

Proof. Assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-filter of X. Then for all $x \in X, A_T(0) \succeq A_T(x), A_I(0) \preceq A_I(x), A_F(0) \succeq A_F(x), \lambda_T(0) \leq \lambda_T(x), \lambda_I(0) \geq \lambda_I(x)$, and $\lambda_F(0) \leq \lambda_F(x)$. Let for all $x, y \in X$. Then

$$A_T(x\cdot y) \succeq \text{rmin}\{A_T(y\cdot(x\cdot y)), A_T(y)\}$$

$$= \text{rmin}\{A_T(0), A_T(y)\} \quad ((3.0.5))$$

$$= A_T(y),$$

$$A_I(x\cdot y) \preceq \text{rmax}\{A_I(y\cdot(x\cdot y)), A_I(y)\}$$

$$= \text{rmax}\{A_I(0), A_I(y)\} \quad ((3.0.5))$$

$$= A_I(y),$$

$$A_F(x\cdot y) \succeq \text{rmin}\{A_F(y\cdot(x\cdot y)), A_F(y)\}$$

$$
\begin{aligned}
&= \text{rmin}\{A_F(0), A_F(y)\} && ((3.0.5))\\
&= A_F(y),\\
\lambda_T(x\cdot y) &\leq \max\{\lambda_T(y\cdot(x\cdot y)), \lambda_T(y)\}\\
&= \max\{\lambda_T(0), \lambda_T(y)\} && ((3.0.5))\\
&= \lambda_T(y),\\
\lambda_I(x\cdot y) &\geq \min\{\lambda_I(y\cdot(x\cdot y)), \lambda_I(y)\}\\
&= \min\{\lambda_I(0), \lambda_I(y)\} && ((3.0.5))\\
&= \lambda_I(y),\\
\lambda_F(x\cdot y) &\leq \max\{\lambda_F(y\cdot(x\cdot y)), \lambda_F(y)\}\\
&= \max\{\lambda_F(0), \lambda_F(y)\} && ((3.0.5))\\
&= \lambda_F(y).
\end{aligned}
$$

Hence, $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic near UP-filter of X. □

The following example show that the converse of Theorem 4.4.18 is not true.

Example 4.4.19 From Example 4.4.5, we have $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic near UP-filter of X. Since $A_T(2) = [0.5, 0.6] \not\succeq [0.6, 0.8] = \text{rmin}\{A_T(1\cdot 2), A_T(1)\}$, we have $\mathscr{A} = (\mathbf{A}, \Lambda)$ is not a neutrosophic cubic UP-filter of X.

Theorem 4.4.20 *Every neutrosophic cubic near UP-filter of X is a neutrosophic cubic UP-subalgebra.*

Proof. Assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic near UP-filter of X. Then for all $x \in X, A_T(0) \succeq A_T(x), A_I(0) \preceq A_I(x), A_F(0) \succeq A_F(x), \lambda_T(0) \leq \lambda_T(x), \lambda_I(0) \geq \lambda_I(x)$, and $\lambda_F(0) \leq \lambda_F(x)$. Let $x, y \in X$. By (2.0.15), we have

$$A_T(x\cdot y) \succeq A_T(y) \succeq \text{rmin}\{A_T(x), A_T(y)\},$$

$$A_I(x \cdot y) \preceq A_I(y) \preceq \text{rmax}\{A_I(x), A_I(y)\},$$

$$A_F(x \cdot y) \succeq A_F(y) \succeq \text{rmin}\{A_F(x), A_F(y)\},$$

$$\lambda_T(x \cdot y) \leq \lambda_T(y) \leq \max\{\lambda_T(x), \lambda_T(y)\},$$

$$\lambda_I(x \cdot y) \geq \lambda_I(y) \geq \min\{\lambda_I(x), \lambda_I(y)\},$$

$$\lambda_F(x \cdot y) \leq \lambda_F(y) \leq \max\{\lambda_F(x), \lambda_F(y)\}.$$

Hence, $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-subalgebra of X. □

The following example show that the converse of Theorem 4.4.20 is not true.

Example 4.4.21 From Example 4.4.3, we have $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-subalgebra of X. Since $\lambda_I(1 \cdot 2) = 0.2 < 0.6 = \lambda_I(2)$, we have $\mathscr{A} = (\mathbf{A}, \Lambda)$ is not a neutrosophic cubic near UP-filter of X.

Theorem 4.4.22 *If $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-subalgebra of X satisfying the following condition:*

$$(\forall x, y \in X) \left(x \cdot y \neq 0 \Rightarrow \begin{cases} A_T(x) \succeq A_T(y) \\ A_I(x) \preceq A_I(y) \\ A_F(x) \succeq A_F(y) \\ \lambda_T(x) \leq \lambda_T(y) \\ \lambda_I(x) \geq \lambda_I(y) \\ \lambda_F(x) \leq \lambda_F(y) \end{cases} \right), \tag{4.4.13}$$

then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic near UP-filter of X.

Proof. Assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-subalgebra of X satisfying the condition (4.4.13). By Proposition 4.4.2, we have $\mathscr{A}$ satisfies the conditions (4.4.3) and (4.4.4). Next, let $x, y \in X$.

Case 1: $x \cdot y = 0$. Then

$$A_T(x \cdot y) = A_T(0) \succeq A_T(y), A_I(x \cdot y) = A_I(0) \preceq A_I(y),$$
$$A_F(x \cdot y) = A_F(0) \succeq A_F(y), \lambda_T(x \cdot y) = \lambda_T(0) \leq \lambda_T(y),$$
$$\lambda_I(x \cdot y) = \lambda_I(0) \geq \lambda_I(y), \lambda_F(x \cdot y) = \lambda_F(0) \leq \lambda_F(y).$$

Case 2: $x \cdot y \neq 0$. Then

$$A_T(x \cdot y) \succeq \text{rmin}\{A_T(x), A_T(y)\} = A_T(y),$$
$$A_I(x \cdot y) \preceq \text{rmax}\{A_I(x), A_I(y)\} = A_I(y),$$
$$A_F(x \cdot y) \succeq \text{rmin}\{A_F(x), A_F(y)\} = A_F(y),$$
$$\lambda_T(x \cdot y) \leq \max\{\lambda_T(x), \lambda_T(y)\} = \lambda_T(y),$$
$$\lambda_I(x \cdot y) \geq \min\{\lambda_I(x), \lambda_I(y)\} = \lambda_I(y),$$
$$\lambda_F(x \cdot y) \leq \max\{\lambda_F(x), \lambda_F(y)\} = \lambda_F(y).$$

Hence, $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic near UP-filter of X. □

Theorem 4.4.23 *If $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic near UP-filter of X satisfying the following condition:*

$$A_T = A_I = A_F, \lambda_T = \lambda_I = \lambda_F, \tag{4.4.14}$$

then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic strong UP-ideal of X.

Proof. Assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic near UP-filter of X satisfying the condition (4.4.14). Then $\mathscr{A}$ satisfies the conditions (4.4.3) and

(4.4.4). Let $x \in X$. Then

$$
\begin{aligned}
A_T(0) \succeq A_T(x) = A_I(x) \succeq A_I(0) = A_T(0)\\
A_I(0) \preceq A_I(x) = A_T(x) \preceq A_T(0) = A_I(0)\\
A_F(0) \succeq A_F(x) = A_I(x) \succeq A_I(0) = A_F(0)\\
\lambda_T(0) \leq \lambda_T(x) = \lambda_I(x) \leq \lambda_I(0) = \lambda_T(0)\\
\lambda_I(x) \geq \lambda_I(x) = \lambda_T(x) \geq \lambda_T(x) = \lambda_I(x)\\
\lambda_F(x) \leq \lambda_F(x) = \lambda_I(x) \leq \lambda_I(x) = \lambda_F(x)
\end{aligned}
$$

Thus $A_T(0) = A_T(x), A_I(0) = A_I(x), A_F(0) = A_F(x), \lambda_T(0) = \lambda_T(x), \lambda_I(x) = \lambda_I(x)$, and $\lambda_F(x) = \lambda_F(x)$, that is, $\mathscr{A}$ is constant. By Theorem 4.4.13, we have $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic strong UP-ideal of X. □

Theorem 4.4.24 *If $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-filter of X satisfying the following condition:*

$$
(\forall x, y, z \in X)\begin{pmatrix}
A_T(y \cdot (x \cdot z)) = A_T(x \cdot (y \cdot z))\\
A_I(y \cdot (x \cdot z)) = A_I(x \cdot (y \cdot z))\\
A_F(y \cdot (x \cdot z)) = A_F(x \cdot (y \cdot z))\\
\lambda_T(y \cdot (x \cdot z)) = \lambda_T(x \cdot (y \cdot z))\\
\lambda_I(y \cdot (x \cdot z)) = \lambda_I(x \cdot (y \cdot z))\\
\lambda_F(y \cdot (x \cdot z)) = \lambda_F(x \cdot (y \cdot z))
\end{pmatrix}, \tag{4.4.15}
$$

then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-ideal of X.

Proof. Assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-filter of X satisfying the condition (4.4.15). Then $\mathscr{A}$ satisfies the conditions (4.4.3) and (4.4.4). Next, let $x, y, z \in X$. Then

$$\begin{aligned}
A_T(x \cdot z) &\succeq \text{rmin}\{A_T(y \cdot (x \cdot z)), A_T(y)\} \\
&= \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\}, \\
A_I(x \cdot z) &\preceq \text{rmax}\{A_I(y \cdot (x \cdot z)), A_I(y)\} \\
&= \text{rmax}\{A_I(x \cdot (y \cdot z)), A_I(y)\}, \\
A_F(x \cdot z) &\succeq \text{rmin}\{A_F(y \cdot (x \cdot z)), A_F(y)\} \\
&= \text{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\}, \\
\lambda_T(x \cdot z) &\leq \max\{\lambda_T(y \cdot (x \cdot z)), \lambda_T(y)\} \\
&= \max\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\}, \\
\lambda_I(x \cdot z) &\geq \min\{\lambda_I(y \cdot (x \cdot z)), \lambda_I(y)\} \\
&= \min\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\}, \\
\lambda_F(x \cdot z) &\leq \max\{\lambda_F(y \cdot (x \cdot z)), \lambda_F(y)\} \\
&= \max\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}.
\end{aligned}$$

Hence, $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-ideal of X. □

Theorem 4.4.25 *If $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a NCS in X satisfying the following condition:*

$$(\forall x, y, z \in X) \left(z \leq x \cdot y \Rightarrow \begin{cases} A_T(z) \succeq \text{rmin}\{A_T(x), A_T(y)\} \\ A_I(z) \preceq \text{rmax}\{A_I(x), A_I(y)\} \\ A_F(z) \succeq \text{rmin}\{A_F(x), A_F(y)\} \\ \lambda_T(z) \leq \max\{\lambda_T(x), \lambda_T(y)\} \\ \lambda_I(z) \geq \min\{\lambda_I(x), \lambda_I(y)\} \\ \lambda_F(z) \leq \max\{\lambda_F(x), \lambda_F(y)\} \end{cases} \right), \tag{4.4.16}$$

then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-subalgebra of X.

Proof. Assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a NCS in X satisfying the condition (4.4.16). Let $x, y \in X$. By (3.0.1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \geq x \cdot y$. It follows from (4.4.16) that

$$A_T(x \cdot y) \succeq \operatorname{rmin}\{A_T(x), A_T(y)\}, A_I(x \cdot y) \preceq \operatorname{rmax}\{A_I(x), A_I(y)\},$$
$$A_F(x \cdot y) \succeq \operatorname{rmin}\{A_F(x), A_F(y)\}, \lambda_T(x \cdot y) \leq \max\{\lambda_T(x), \lambda_T(y)\},$$
$$\lambda_I(x \cdot y) \geq \min\{\lambda_I(x), \lambda_I(y)\}, \lambda_F(x \cdot y) \leq \max\{\lambda_F(x), \lambda_F(y)\}.$$

Hence, $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-subalgebra of X. □

Theorem 4.4.26 *If $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a NCS in X satisfying the following condition:*

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} A_T(y) \succeq \operatorname{rmin}\{A_T(z), A_T(x)\} \\ A_I(y) \preceq \operatorname{rmax}\{A_I(z), A_I(x)\} \\ A_F(y) \succeq \operatorname{rmin}\{A_F(z), A_F(x)\} \\ \lambda_T(y) \leq \max\{\lambda_T(z), \lambda_T(x)\} \\ \lambda_I(y) \geq \min\{\lambda_I(z), \lambda_I(x)\} \\ \lambda_F(y) \leq \max\{\lambda_F(z), \lambda_F(x)\} \end{cases} \right), \quad (4.4.17)$$

then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-filter of X.

Proof. Assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a NCS in X satisfying the condition (4.4.17). Let $x \in X$. By (UP-3), we have $x \cdot (x \cdot 0) = 0$, that is, $x \leq x \cdot 0$. It follows from (4.4.17) that

$$A_T(0) \succeq \operatorname{rmin}\{A_T(x), A_T(x)\} = A_T(x),$$
$$A_I(0) \preceq \operatorname{rmax}\{A_I(x), A_I(x)\} = A_I(x),$$
$$A_F(0) \succeq \operatorname{rmin}\{A_F(x), A_F(x)\} = A_F(x),$$
$$\lambda_T(0) \leq \max\{\lambda_T(x), \lambda_T(x)\} = \lambda_T(x),$$

$$\lambda_I(0) \geq \min\{\lambda_I(x), \lambda_I(x)\} = \lambda_I(x),$$
$$\lambda_F(0) \leq \max\{\lambda_F(x), \lambda_F(x)\} = \lambda_F(x).$$

Next, let $x, y \in X$. By (3.0.1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \geq x \cdot y$. It follows from (4.4.17) that

$$A_T(y) \succeq \text{rmin}\{A_T(x \cdot y), A_T(x)\}, A_I(y) \preceq \text{rmax}\{A_I(x \cdot y), A_I(x)\},$$
$$A_F(y) \succeq \text{rmin}\{A_F(x \cdot y), A_F(x)\}, \lambda_T(y) \leq \max\{\lambda_T(x \cdot y), \lambda_T(x)\},$$
$$\lambda_I(y) \geq \min\{\lambda_I(x \cdot y), \lambda_I(x)\}, \lambda_F(y) \leq \max\{\lambda_F(x \cdot y), \lambda_F(x)\}.$$

Hence, $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-filter of X. □

Theorem 4.4.27 *If $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a NCS in X satisfying the following condition:*

$$(\forall a, x, y, z \in X) \left(a \leq x \cdot (y \cdot z) \Rightarrow \begin{cases} A_T(x \cdot z) \succeq \text{rmin}\{A_T(a), A_T(y)\} \\ A_I(x \cdot z) \preceq \text{rmax}\{A_I(a), A_I(y)\} \\ A_F(x \cdot z) \succeq \text{rmin}\{A_F(a), A_F(y)\} \\ \lambda_T(x \cdot z) \leq \max\{\lambda_T(a), \lambda_T(y)\} \\ \lambda_I(x \cdot z) \geq \min\{\lambda_I(a), \lambda_I(y)\} \\ \lambda_F(x \cdot z) \leq \max\{\lambda_F(a), \lambda_F(y)\} \end{cases} \right), \tag{4.4.18}$$

then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-ideal of X.

Proof. Assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a NCS in X satisfying the condition (4.4.18). Let $x \in X$. By (UP-3), we have $x \cdot (0 \cdot (x \cdot 0)) = 0$, that is, $x \leq 0 \cdot (x \cdot 0)$. It follows from (4.4.18) that

$$A_T(0) = A_T(0 \cdot 0) \succeq \text{rmin}\{A_T(x), A_T(x)\} = A_T(x), \qquad \text{((UP-2))}$$
$$A_I(0) = A_I(0 \cdot 0) \preceq \text{rmax}\{A_I(x), A_I(x)\} = A_I(x), \qquad \text{((UP-2))}$$

$$A_F(0) = A_F(0 \cdot 0) \succeq \text{rmin}\{A_F(x), A_F(x)\} = A_F(x), \qquad \text{((UP-2))}$$

$$\lambda_T(0) = \lambda_T(0 \cdot 0) \leq \max\{\lambda_T(x), \lambda_T(x)\} = \lambda_T(x), \qquad \text{((UP-2))}$$

$$\lambda_I(0) = \lambda_I(0 \cdot 0) \geq \min\{\lambda_I(x), \lambda_I(x)\} = \lambda_I(x), \qquad \text{((UP-2))}$$

$$\lambda_F(0) = \lambda_F(0 \cdot 0) \leq \max\{\lambda_F(x), \lambda_F(x)\} = \lambda_F(x). \qquad \text{((UP-2))}$$

Next, let $x, y, z \in X$. By (3.0.1), we have $(x \cdot (y \cdot z)) \cdot (x \cdot (y \cdot z)) = 0$, that is, $x \cdot (y \cdot z) \geq x \cdot (y \cdot z)$. It follows from (4.4.18) that

$$A_T(x \cdot z) \succeq \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\},$$

$$A_I(x \cdot z) \preceq \text{rmax}\{A_I(x \cdot (y \cdot z)), A_I(y)\},$$

$$A_F(x \cdot z) \succeq \text{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\},$$

$$\lambda_T(x \cdot z) \leq \max\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\},$$

$$\lambda_I(x \cdot z) \geq \min\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\},$$

$$\lambda_F(x \cdot z) \leq \max\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}.$$

Hence, $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-ideal of X. □

Theorem 4.4.28 *A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X satisfies the following condition:*

$$(\forall x, y, z \in X) \left(z \leq x \cdot y \Rightarrow \begin{cases} A_T(z) \succeq A_T(y) \\ A_I(z) \preceq A_I(y) \\ A_F(z) \succeq A_F(y) \\ \lambda_T(z) \leq \lambda_T(y) \\ \lambda_I(z) \geq \lambda_I(y) \\ \lambda_F(z) \leq \lambda_F(y) \end{cases} \right) \qquad (4.4.19)$$

if and only if $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic strong UP-ideal of X.

Proof. Assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a NCS in X satisfying the condition (4.4.19). Let $x, y \in X$. By (UP-3) and (3.0.1), we have $x \cdot 0 = 0$, that is, $x \leq 0 = y \cdot y$. It follows from (4.4.19) that

$$\begin{aligned} A_T(x) \succeq A_T(y), A_I(x) \preceq A_I(y), A_F(x) \succeq A_F(y),\\ \lambda_T(x) \leq \lambda_T(y), \lambda_I(x) \geq \lambda_I(y), \lambda_F(x) \leq \lambda_F(y).\end{aligned}$$

Similarly,

$$\begin{aligned} A_T(y) \succeq A_T(x), A_I(y) \preceq A_I(x), A_F(y) \succeq A_F(x),\\ \lambda_T(y) \leq \lambda_T(x), \lambda_I(y) \geq \lambda_I(x), \lambda_F(y) \leq \lambda_F(x).\end{aligned}$$

Then

$$\begin{aligned} A_T(x) = A_T(y), A_I(x) = A_I(y), A_F(x) = A_F(y),\\ \lambda_T(x) = \lambda_T(y), \lambda_I(x) = \lambda_I(y), \lambda_F(x) = \lambda_F(y).\end{aligned}$$

Thus $\mathscr{A}$ is constant. By Theorem 4.4.13, we have $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic strong UP-ideal of X. □

Then, we have the diagram of generalization of NCSs in UP-algebras as shown in Figure 4.4.

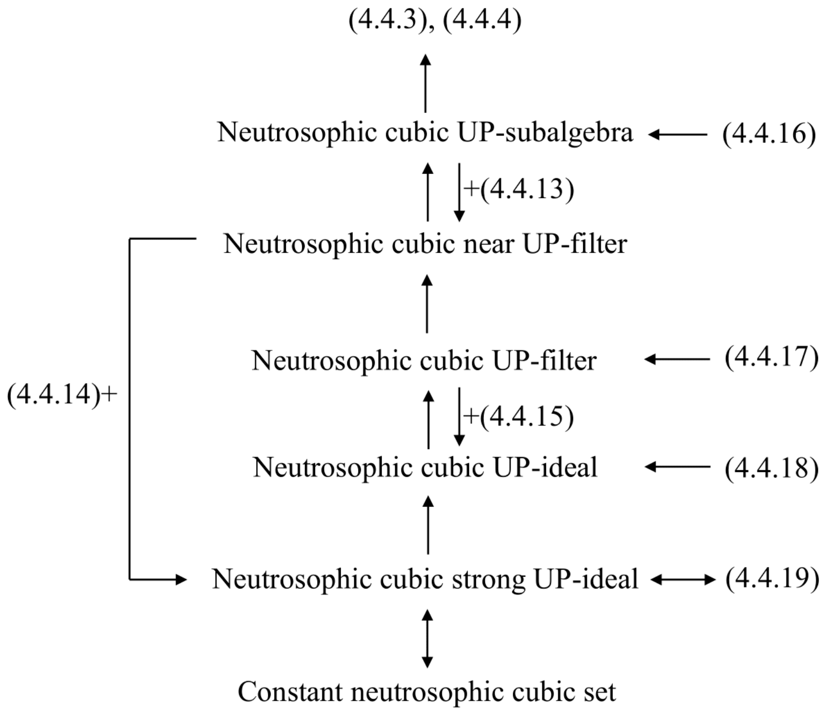

Figure 4.4: Neutrosophic cubic sets in UP-algebras

From the definitions of the NS ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ in Section 4.2 and the IVNS $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ in Section 4.3, we will define the NCS $\mathscr{A}^G[[\tilde{a},\tilde{b},\tilde{c}],[\alpha,\beta,\gamma]]$.

For any fixed numbers $\alpha^+,\alpha^-,\beta^+,\beta^-,\gamma^+,\gamma^- \in [0,1]$ such that $\alpha^+ > \alpha^-, \beta^+ > \beta^-, \gamma^+ > \gamma^-$, for any fixed interval numbers $\tilde{a}^+,\tilde{a}^-,\tilde{b}^+,\tilde{b}^-,\tilde{c}^+,\tilde{c}^- \in [[0,1]]$ such that $\tilde{a}^+ \succ \tilde{a}^-, \tilde{b}^+ \succ \tilde{b}^-, \tilde{c}^+ \succ \tilde{c}^-$, and a nonempty subset G of X, we define the NCS $\mathscr{A}^G[[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}],[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]] = (\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}], {}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}])$ in X.

Combining Theorems 4.4.12, 4.2.29 - 4.2.33, and 4.1.31 - 4.1.35, we have the following corollary.

Corollary 4.4.29 *A NCS $\mathscr{A}^G[[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}],[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]]$ in X is a neutrosophic cubic UP-subalgebra (resp., neutrosophic cubic near UP-filter, neutrosophic cubic UP-filter, neutrosophic cubic UP-ideal, neutrosophic cubic strong UP-ideal) of X if and only if a nonempty subset G of X is a UP-subalgebra (resp., near UP-filter,* *UP-filter, UP-ideal, strong UP-ideal) of X.*

Next, we discuss the relationships among neutrosophic cubic UP-subalgebras (resp., neutrosophic cubic near UP-filters, neutrosophic cubic UP-filters, neutrosophic cubic UP-ideals, neutrosophic cubic strong UP-ideals) of UP-algebras and their level subsets.

Combining Theorems 4.4.12, 4.2.34 - 4.2.37, and 4.3.37 - 4.3.40, we have the following corollary.

Corollary 4.4.30 *A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X is a neutrosophic cubic UP-subalgebra (resp., neutrosophic cubic near UP-filter, neutrosophic cubic UP-filter, neutrosophic cubic UP-ideal) of X if and only if for all $[s_{T_1}, s_{T_2}], [s_{I_1}, s_{I_2}], [s_{F_1}, s_{F_2}] \in [[0, 1]]$ and $t_T, t_I, t_F \in [0, 1]$, the sets $U(A_T; [s_{T_1}, s_{T_2}]), L(A_I; [s_{I_1}, s_{I_2}]),$ $U(A_F; [s_{F_1}, s_{F_2}]), L(\lambda_T; t_T), U(\lambda_I; t_I)$, and $L(\lambda_F; t_F)$ are either empty or UP-subalgebras (resp., near UP-filter, UP-filter, UP-ideal) of X.*

Combining Theorems 4.4.12, 4.1.47, and 4.3.41, we have the following corollary.

Corollary 4.4.31 *A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X is a neutrosophic cubic strong UP-ideal of X if and only if the sets $E(A_T; A_T(0)), E(A_I; A_I(0)), E(A_F; A_F(0)),$ $E(\lambda_T, \lambda_T(0)), E(\lambda_I, \lambda_I(0))$, and $E(\lambda_F, \lambda_F(0))$ are strong UP-ideals of X.*

4.5 Homomorphism of neutrosophic cubic sets in UP-algebras

In this section, the image and inverse image of neutrosophic cubic set are defined and some results are studied.

Definition 4.5.1 Let f be mapping from a nonempty set X into a nonempty set Y and $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ be a NCS in X. Then the image of $\mathscr{A}$ under f is defined as a NCS $f(\mathscr{A}) = (f(A)_{T,I,F}, f(\lambda)_{T,I,F})$ in Y, where

$$f(A)_T(y) = \begin{cases} \operatorname{rsup}_{x\in f^{-1}(y)}\{A_T(x)\} & \text{if } f^{-1}(y) \neq \emptyset, \\ [0,0] & \text{otherwise,} \end{cases} \tag{4.5.1}$$

$$f(A)_I(y) = \begin{cases} \operatorname{rinf}_{x\in f^{-1}(y)}\{A_I(x)\} & \text{if } f^{-1}(y) \neq \emptyset, \\ [1,1] & \text{otherwise,} \end{cases} \tag{4.5.2}$$

$$f(A)_F(y) = \begin{cases} \operatorname{rsup}_{x\in f^{-1}(y)}\{A_F(x)\} & \text{if } f^{-1}(y) \neq \emptyset, \\ [0,0] & \text{otherwise,} \end{cases} \tag{4.5.3}$$

$$f(\lambda)_T(y) = \begin{cases} \inf_{x\in f^{-1}(y)}\{\lambda_T(x)\} & \text{if } f^{-1}(y) \neq \emptyset, \\ 1 & \text{otherwise,} \end{cases} \tag{4.5.4}$$

$$f(\lambda)_I(y) = \begin{cases} \sup_{x\in f^{-1}(y)}\{\lambda_I(x)\} & \text{if } f^{-1}(y) \neq \emptyset, \\ 0 & \text{otherwise,} \end{cases} \tag{4.5.5}$$

$$f(\lambda)_F(y) = \begin{cases} \inf_{x\in f^{-1}(y)}\{\lambda_F(x)\} & \text{if } f^{-1}(y) \neq \emptyset, \\ 1 & \text{otherwise.} \end{cases} \tag{4.5.6}$$

Example 4.5.2 Let $X = \{0_X, 1_X, 2_X\}$ be a UP-algebra with a fixed element 0_X and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0_X	1_X	2_X
0_X	0_X	1_X	2_X
1_X	0_X	0_X	1_X
2_X	0_X	0_X	0_X

and let $Y = \{0_Y, 1_Y, 2_Y\}$ be a UP-algebra with a fixed element 0_Y and a binary operation $*$ defined by the following Cayley table:

$*$	0_Y	1_Y	2_Y
0_Y	0_Y	1_Y	2_Y
1_Y	0_Y	0_Y	2_Y
2_Y	0_Y	0_Y	0_Y

We define a mapping $f : X \to Y$ as follows:

$$f(0_X) = 0_Y, f(1_X) = 1_Y, \text{ and } f(2_X) = 1_Y.$$

We define a NCS $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ in X with the tabular representation as follows:

X	$\mathbf{A}(x)$	$\Lambda(x)$
0_X	$([0.4, 0.7], [0.5, 0.7], [0.2, 0.4])$	$(0.1, 0.3, 0.4)$
1_X	$([0.1, 0.2], [0.1, 0.5], [0.4, 0.5])$	$(0.3, 0.8, 0.4)$
2_X	$([0.8, 0.9], [0.7, 0.8], [0.1, 0.6])$	$(0.1, 0.5, 0.7)$

Then $f(\mathscr{A}) = (f(A)_{T,I,F}, f(\lambda)_{T,I,F})$ in Y with the tabular representation as follows:

Y	$\mathbf{A}(x)$	$\Lambda(x)$
0_Y	$([0.4, 0.7], [0.5, 0.7], [0.2, 0.4])$	$(0.1, 0.3, 0.4)$
1_Y	$([0.8, 0.9], [0.1, 0.5], [0.4, 0.6])$	$(0.1, 0.8, 0.4)$
2_Y	$([0, 0], [1, 1], [0, 0])$	$(1, 0, 1)$

Hence, $f(\mathscr{A}) = (f(A)_{T,I,F}, f(\lambda)_{T,I,F})$ is a NCS in Y.

Definition 4.5.3 Let f be mapping from a nonempty set X into a nonempty set Y and $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ be a NCS in Y. Then the inverse image of $\mathscr{A}$ is defined as a NCS $f^{-1}(\mathscr{A}) = (f^{-1}(A)_{T,I,F}, f^{-1}(\lambda)_{T,I,F})$ in X, where

$$(\forall x \in X)(f^{-1}(A)_{T,I,F}(x) = A_{T,I,F}(f(x))), \tag{4.5.7}$$

$$(\forall x \in X)(f^{-1}(\lambda)_{T,I,F}(x) = \lambda_{T,I,F}(f(x))). \tag{4.5.8}$$

Example 4.5.4 In Example 4.5.2, we have $(X, \cdot, 0_X)$ and $(Y, *, 0_Y)$ are UP-algebras. We define a mapping $f : X \to Y$ as follows:

$$f(0_X) = 0_Y, f(1_X) = 1_Y, \text{ and } f(2_X) = 1_Y.$$

We define a NCS $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ in Y with the tabular representation as follows:

Y	$\mathbf{A}(x)$	$\Lambda(x)$
0_Y	$([0.3, 0.7], [0.3, 0.5], [0.1, 0.4])$	$(0.5, 0.4, 0.7)$
1_Y	$([0.6, 0.7], [0.1, 0.3], [0.4, 0.5])$	$(0.2, 0.7, 0.8)$
2_Y	$([0.5, 0.9], [0.3, 0.5], [0.5, 0.8])$	$(0.3, 0.5, 0.4)$

Then $f^{-1}(\mathscr{A}) = (f^{-1}(A)_{T,I,F}, f^{-1}(\lambda)_{T,I,F})$ in X with the tabular representation as follows:

X	$\mathbf{A}(x)$	$\Lambda(x)$
0_X	$([0.3, 0.7], [0.3, 0.5], [0.1, 0.4])$	$(0.5, 0.4, 0.7)$
1_X	$([0.6, 0.7], [0.1, 0.3], [0.4, 0.5])$	$(0.2, 0.7, 0.8)$
2_X	$([0.6, 0.7], [0.1, 0.3], [0.4, 0.5])$	$(0.2, 0.7, 0.8)$

Hence, $f^{-1}(\mathscr{A}) = (f^{-1}(A)_{T,I,F}, f^{-1}(\lambda)_{T,I,F})$ is a NCS in X.

Definition 4.5.5 A NCS $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ in X is said to be *order preserving* if

$$(\forall x, y \in X)\left(x \leq y \Rightarrow \begin{cases} A_T(x) \preceq A_T(y), A_I(x) \succeq A_I(y), A_F(x) \preceq A_F(y), \\ \lambda_T(x) \geq \lambda_T(y), \lambda_I(x) \leq \lambda_I(y), \lambda_F(x) \geq \lambda_F(y) \end{cases}\right). \tag{4.5.9}$$

Lemma 4.5.6 *Every neutrosophic cubic UP-filter (resp., neutrosophic cubic UP-ideal, neutrosophic cubic strong UP-ideal) of X is order preserving.*

Proof. Assume that $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ is a neutrosophic cubic UP-filter of X. Let $x, y \in X$ be such that $x \leq y$ in X. Then $x \cdot y = 0$. Thus

$$\begin{aligned}
A_T(y) &\succeq \operatorname{rmin}\{A_T(x \cdot y), A_T(x)\} && ((4.4.7))\\
&= \operatorname{rmin}\{A_T(0), A_T(x)\}\\
&= A_T(x), && ((4.4.3),(2.0.23))\\
A_I(y) &\preceq \operatorname{rmax}\{A_I(x \cdot y), A_I(x)\} && ((4.4.7))\\
&= \operatorname{rmin}\{A_I(0), A_I(x)\}\\
&= A_I(x), && ((4.4.3),(2.0.24))\\
A_F(y) &\succeq \operatorname{rmin}\{A_F(x \cdot y), A_F(x)\} && ((4.4.7))\\
&= \operatorname{rmin}\{A_F(0), A_F(x)\}\\
&= A_F(x), && ((4.4.3),(2.0.23))\\
\lambda_T(y) &\leq \max\{\lambda_T(x \cdot y), \lambda_T(x)\} && ((4.4.8))\\
&= \max\{\lambda_T(0), \lambda_T(x)\}\\
&= \lambda_T(x), && ((4.4.4))\\
\lambda_I(y) &\geq \min\{\lambda_I(x \cdot y), \lambda_I(x)\} && ((4.4.8))\\
&= \min\{\lambda_I(0), \lambda_I(x)\}\\
&= \lambda_I(x), && ((4.4.4))\\
\lambda_F(y) &\leq \max\{\lambda_F(x \cdot y), \lambda_F(x)\} && ((4.4.8))\\
&= \max\{\lambda_F(0), \lambda_F(x)\}\\
&= \lambda_F(x). && ((4.4.4))
\end{aligned}$$

Hence, $\mathscr{A}$ is order preserving. □

Theorem 4.5.7 *Let $(X, \cdot, 0_X)$ and $(Y, *, 0_Y)$ be UP-algebras, $f\colon X \to Y$ be a UP-homomorphism, and $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ be a NCS in Y. Then the following statements hold:*

(1) *If* $\mathscr{A}$ *is a neutrosophic cubic UP-subalgebra of* Y*, then the inverse image* $f^{-1}(\mathscr{A})$ *of* $\mathscr{A}$ *under* f *is a neutrosophic cubic UP-subalgebra of* X*.*

(2) *If* $\mathscr{A}$ *is a neutrosophic cubic near UP-filter of* Y *which is order preserving, then the inverse image* $f^{-1}(\mathscr{A})$ *of* $\mathscr{A}$ *under* f *is a neutrosophic cubic near UP-filter of* X*.*

(3) *If* $\mathscr{A}$ *is a neutrosophic cubic UP-filter of* Y*, then the inverse image* $f^{-1}(\mathscr{A})$ *of* $\mathscr{A}$ *under* f *is a neutrosophic cubic UP-filter of* X*.*

(4) *If* $\mathscr{A}$ *is a neutrosophic cubic UP-ideal of* Y*, then the inverse image* $f^{-1}(\mathscr{A})$ *of* $\mathscr{A}$ *under* f *is a neutrosophic cubic UP-ideal of* X*.*

(5) *If* $\mathscr{A}$ *is a neutrosophic cubic strong UP-ideal of* Y*, then the inverse image* $f^{-1}(\mathscr{A})$ *of* $\mathscr{A}$ *under* f *is a neutrosophic cubic strong UP-ideal of* X*.*

Proof. (1) Assume that $\mathscr{A}$ is a neutrosophic cubic UP-subalgebra of Y. Then for all $x, y \in X$,

$$\begin{aligned}
f^{-1}(A)_T(x \cdot y) &= A_T(f(x \cdot y)) && ((4.5.7))\\
&= A_T(f(x) * f(y))\\
&\succeq \text{rmin}\{A_T(f(x)), A_T(f(y))\} && ((4.4.1))\\
&= \text{rmin}\{f^{-1}(A)_T(x), f^{-1}(A)_T(y)\}, && ((4.5.7))\\
f^{-1}(A)_I(x \cdot y) &= A_I(f(x \cdot y)) && ((4.5.7))\\
&= A_I(f(x) * f(y))\\
&\preceq \text{rmax}\{A_I(f(x)), A_I(f(y))\} && ((4.4.1))\\
&= \text{rmax}\{f^{-1}(A)_I(x), f^{-1}(A)_I(y)\}, && ((4.5.7))\\
f^{-1}(A)_F(x \cdot y) &= A_F(f(x \cdot y)) && ((4.5.7))\\
&= A_F(f(x) * f(y))\\
&\succeq \text{rmin}\{A_F(f(x)), A_F(f(y))\} && ((4.4.1))
\end{aligned}$$

$$
\begin{aligned}
&= \operatorname{rmin}\{f^{-1}(A)_F(x), f^{-1}(A)_F(y)\}, && ((4.5.7))\\
f^{-1}(\lambda)_T(x \cdot y) &= \lambda_T(f(x \cdot y)) && ((4.5.8))\\
&= \lambda_T(f(x) * f(y))\\
&\leq \max\{\lambda_T(f(x)), \lambda_T(f(y))\} && ((4.4.2))\\
&= \max\{f^{-1}(\lambda)_T(x), f^{-1}(\lambda)_T(y)\}, && ((4.5.8))\\
f^{-1}(\lambda)_I(x \cdot y) &= \lambda_I(f(x \cdot y)) && ((4.5.8))\\
&= \lambda_I(f(x) * f(y))\\
&\geq \min\{\lambda_I(f(x)), \lambda_I(f(y))\} && ((4.4.2))\\
&= \min\{f^{-1}(\lambda)_I(x), f^{-1}(\lambda)_I(y)\}, && ((4.5.8))\\
f^{-1}(\lambda)_F(x \cdot y) &= \lambda_F(f(x \cdot y)) && ((4.5.8))\\
&= \lambda_F(f(x) * f(y))\\
&\leq \max\{\lambda_F(f(x)), \lambda_F(f(y))\} && ((4.4.2))\\
&= \max\{f^{-1}(\lambda)_F(x), f^{-1}(\lambda)_F(y)\}. && ((4.5.8))
\end{aligned}
$$

Hence, $f^{-1}(\mathscr{A})$ is a neutrosophic cubic UP-subalgebra of X.

(2) Assume that $\mathscr{A}$ is a neutrosophic cubic near UP-filter of Y which is order preserving. By Theorem 3.0.8 (2) and (UP-3), we have for all $x \in X$,

$$
\begin{aligned}
f^{-1}(A)_T(0_X) = A_T(f(0_X)) \succeq A_T(f(x)) = f^{-1}(A)_T(x),\\
f^{-1}(A)_I(0_X) = A_I(f(0_X)) \preceq A_I(f(x)) = f^{-1}(A)_I(x),\\
f^{-1}(A)_F(0_X) = A_F(f(0_X)) \succeq A_F(f(x)) = f^{-1}(A)_F(x),\\
f^{-1}(\lambda)_T(0_X) = \lambda_T(f(0_X)) \leq \lambda_T(f(x)) = f^{-1}(\lambda)_T(x),\\
f^{-1}(\lambda)_I(0_X) = \lambda_I(f(0_X)) \geq \lambda_I(f(x)) = f^{-1}(\lambda)_I(x),\\
f^{-1}(\lambda)_F(0_X) = \lambda_F(f(0_X)) \leq \lambda_F(f(x)) = f^{-1}(\lambda)_F(x).
\end{aligned}
$$

Let $x, y \in X$. Then

$$
\begin{aligned}
f^{-1}(A)_T(x \cdot y) &= A_T(f(x \cdot y)) && ((4.5.7))\\
&= A_T(f(x) * f(y))\\
&\succeq A_T(f(y)) && ((4.4.5))\\
&= f^{-1}(A)_T(y), && ((4.5.7))\\
f^{-1}(A)_I(x \cdot y) &= A_I(f(x \cdot y)) && ((4.5.7))\\
&= A_I(f(x) * f(y))\\
&\preceq A_I(f(y)) && ((4.4.5))\\
&= f^{-1}(A)_I(y), && ((4.5.7))\\
f^{-1}(A)_F(x \cdot y) &= A_F(f(x \cdot y)) && ((4.5.7))\\
&= A_F(f(x) * f(y))\\
&\succeq A_F(f(y)) && ((4.4.5))\\
&= f^{-1}(A)_F(y), && ((4.5.7))\\
f^{-1}(\lambda)_T(x \cdot y) &= \lambda_T(f(x \cdot y)) && ((4.5.8))\\
&= \lambda_T(f(x) * f(y))\\
&\leq \lambda_T(f(y)) && ((4.4.6))\\
&= f^{-1}(\lambda)_T(y), && ((4.5.8))\\
f^{-1}(\lambda)_I(x \cdot y) &= \lambda_I(f(x \cdot y)) && ((4.5.8))\\
&= \lambda_I(f(x) * f(y))\\
&\geq \lambda_I(f(y)) && ((4.4.6))\\
&= f^{-1}(\lambda)_I(y), && ((4.5.8))\\
f^{-1}(\lambda)_F(x \cdot y) &= \lambda_F(f(x \cdot y)) && ((4.5.8))\\
&= \lambda_F(f(x) * f(y))\\
&\leq \lambda_F(f(y)) && ((4.4.6))
\end{aligned}
$$

$$= f^{-1}(\lambda)_F(y). \quad ((4.5.8))$$

Hence, $f^{-1}(\mathscr{A})$ is a neutrosophic cubic near UP-filter of X.

(3) Assume that $\mathscr{A}$ is a neutrosophic cubic UP-filter of Y. Then $\mathscr{A}$ is a neutrosophic cubic near UP-filter of Y. By Lemma 4.5.6 and the proof of (2), we have $f^{-1}(\mathscr{A})$ satisfies the conditions (4.4.3) and (4.4.4). Let $x, y \in X$. Then

$$\begin{aligned}
f^{-1}(A)_T(y) &= A_T(f(y)) && ((4.5.7))\\
&\succeq \text{rmin}\{A_T(f(x) * f(y)), A_T(f(x))\} && ((4.4.7))\\
&= \text{rmin}\{A_T(f(x \cdot y)), A_T(f(x))\}\\
&= \text{rmin}\{f^{-1}(A)_T(x \cdot y), f^{-1}(A)_T(x)\}, && ((4.5.7))\\
f^{-1}(A)_I(y) &= A_I(f(y)) && ((4.5.7))\\
&\preceq \text{rmax}\{A_I(f(x) * f(y)), A_I(f(x))\} && ((4.4.7))\\
&= \text{rmax}\{A_I(f(x \cdot y)), A_I(f(x))\}\\
&= \text{rmax}\{f^{-1}(A)_I(x \cdot y), f^{-1}(A)_I(x)\}, && ((4.5.7))\\
f^{-1}(A)_F(y) &= A_F(f(y)) && ((4.5.7))\\
&\succeq \text{rmin}\{A_F(f(x) * f(y)), A_F(f(x))\} && ((4.4.7))\\
&= \text{rmin}\{A_F(f(x \cdot y)), A_F(f(x))\}\\
&= \text{rmin}\{f^{-1}(A)_F(x \cdot y), f^{-1}(A)_F(x)\}, && ((4.5.7))\\
f^{-1}(\lambda)_T(y) &= \lambda_T(f(y)) && ((4.5.8))\\
&\leq \max\{\lambda_T(f(x) * f(y)), \lambda_T(f(x))\} && ((4.4.8))\\
&= \max\{\lambda_T(f(x \cdot y)), \lambda_T(f(x))\}\\
&= \max\{f^{-1}(\lambda)_T(x \cdot y), f^{-1}(\lambda)_T(x)\}, && ((4.5.8))\\
f^{-1}(\lambda)_I(y) &= \lambda_I(f(y)) && ((4.5.8))\\
&\geq \min\{\lambda_I(f(x) * f(y)), \lambda_I(f(x))\} && ((4.4.8))
\end{aligned}$$

$$= \min\{\lambda_I(f(x \cdot y)), \lambda_I(f(x))\}$$

$$= \min\{f^{-1}(\lambda)_I(x \cdot y), f^{-1}(\lambda)_I(x)\}, \quad ((4.5.8))$$

$$f^{-1}(\lambda)_F(y) = \lambda_F(f(y)) \quad ((4.5.8))$$

$$\leq \max\{\lambda_F(f(x) * f(y)), \lambda_F(f(x))\} \quad ((4.4.8))$$

$$= \max\{\lambda_F(f(x \cdot y)), \lambda_F(f(x))\}$$

$$= \max\{f^{-1}(\lambda)_F(x \cdot y), f^{-1}(\lambda)_F(x)\}. \quad ((4.5.8))$$

Hence, $f^{-1}(\mathscr{A})$ is a neutrosophic cubic UP-filter of X.

(4) Assume that $\mathscr{A}$ is a neutrosophic cubic UP-ideal of Y. Then $\mathscr{A}$ is a neutrosophic cubic UP-filter of Y. By the proof of (3), we have $f^{-1}(\mathscr{A})$ satisfies the conditions (4.4.3) and (4.4.4). Let $x, y, z \in X$. Then

$$f^{-1}(A)_T(x \cdot z) = A_T(f(x \cdot z)) \quad ((4.5.7))$$

$$= A_T(f(x) * f(z))$$

$$\succeq \text{rmin}\{A_T(f(x) * (f(y) * f(z))), A_T(f(y))\} \quad ((4.4.9))$$

$$= \text{rmin}\{A_T(f(x) * (f(y \cdot z))), A_T(f(y))\}$$

$$= \text{rmin}\{A_T(f(x \cdot (y \cdot z))), A_T(f(y))\}$$

$$= \text{rmin}\{f^{-1}(A)_T(x \cdot (y \cdot z)), f^{-1}(A)_T(y)\}, \quad ((4.5.7))$$

$$f^{-1}(A)_I(x \cdot z) = A_I(f(x \cdot z)) \quad ((4.5.7))$$

$$= A_I(f(x) * f(z))$$

$$\preceq \text{rmax}\{A_I(f(x) * (f(y) * f(z))), A_I(f(y))\} \quad ((4.4.9))$$

$$= \text{rmax}\{A_I(f(x) * (f(y \cdot z))), A_I(f(y))\}$$

$$= \text{rmax}\{A_I(f(x \cdot (y \cdot z))), A_I(f(y))\}$$

$$= \text{rmax}\{f^{-1}(A)_I(x \cdot (y \cdot z)), f^{-1}(A)_I(y)\}, \quad ((4.5.7))$$

$$f^{-1}(A)_F(x \cdot z) = A_F(f(x \cdot z)) \quad ((4.5.7))$$

$$
\begin{aligned}
&= A_F(f(x) * f(z)) \\
&\succeq \text{rmin}\{A_F(f(x) * (f(y) * f(z))), A_F(f(y))\} && ((4.4.9)) \\
&= \text{rmin}\{A_F(f(x) * (f(y \cdot z))), A_F(f(y))\} \\
&= \text{rmin}\{A_F(f(x \cdot (y \cdot z))), A_F(f(y))\} \\
&= \text{rmin}\{f^{-1}(A)_F(x \cdot (y \cdot z)), f^{-1}(A)_F(y)\}, && ((4.5.7)) \\
f^{-1}(\lambda)_T(x \cdot z) &= \lambda_T(f(x \cdot z)) && ((4.5.8)) \\
&= \lambda_T(f(x) * f(z)) \\
&\leq \max\{\lambda_T(f(x) * (f(y) * f(z))), \lambda_T(f(y))\} && ((4.4.10)) \\
&= \max\{\lambda_T(f(x) * (f(y \cdot z))), \lambda_T(f(y))\} \\
&= \max\{\lambda_T(f(x \cdot (y \cdot z))), \lambda_T(f(y))\} \\
&= \max\{f^{-1}(\lambda)_T(x \cdot (y \cdot z)), f^{-1}(\lambda)_T(y)\}, && ((4.5.8)) \\
f^{-1}(\lambda)_I(x \cdot z) &= \lambda_I(f(x \cdot z)) && ((4.5.8)) \\
&= \lambda_I(f(x) * f(z)) \\
&\geq \min\{\lambda_I(f(x) * (f(y) * f(z))), \lambda_I(f(y))\} && ((4.4.10)) \\
&= \min\{\lambda_I(f(x) * (f(y \cdot z))), \lambda_I(f(y))\} \\
&= \min\{\lambda_I(f(x \cdot (y \cdot z))), \lambda_I(f(y))\} \\
&= \min\{f^{-1}(\lambda)_I(x \cdot (y \cdot z)), f^{-1}(\lambda)_I(y)\}, && ((4.5.8)) \\
f^{-1}(\lambda)_F(x \cdot z) &= \lambda_F(f(x \cdot z)) && ((4.5.8)) \\
&= \lambda_F(f(x) * f(z)) \\
&\leq \max\{\lambda_F(f(x) * (f(y) * f(z))), \lambda_F(f(y))\} && ((4.4.10)) \\
&= \max\{\lambda_F(f(x) * (f(y \cdot z))), \lambda_F(f(y))\} \\
&= \max\{\lambda_F(f(x \cdot (y \cdot z))), \lambda_F(f(y))\} \\
&= \max\{f^{-1}(\lambda)_F(x \cdot (y \cdot z)), f^{-1}(\lambda)_F(y)\}. && ((4.5.8))
\end{aligned}
$$

Hence, $f^{-1}(\mathscr{A})$ is a neutrosophic cubic UP-ideal of X.

(5) Assume that $\mathscr{A}$ is a neutrosophic cubic strong UP-ideal of Y. Then $\mathscr{A}$ is a neutrosophic cubic UP-ideal of Y. By the proof of (4), we have $f^{-1}(\mathscr{A})$ satisfies the conditions (4.4.3) and (4.4.4). Let $x, y, z \in X$. Then

$$
\begin{aligned}
f^{-1}(A)_T(x) &= A_T(f(x)) && ((4.5.7))\\
&\succeq \text{rmin}\{A_T((f(z) * f(y)) * (f(z) * f(x))), A_T(f(y))\} && ((4.4.11))\\
&= \text{rmin}\{A_T(f(z \cdot y) * f(z \cdot x)), A_T(f(y))\}\\
&= \text{rmin}\{A_T(f((z \cdot y) \cdot (z \cdot x))), A_T(f(y))\}\\
&= \text{rmin}\{f^{-1}(A)_T((z \cdot y) \cdot (z \cdot x)), f^{-1}(A)_T(y)\}, && ((4.5.7))\\
f^{-1}(A)_I(x) &= A_I(f(x)) && ((4.5.7))\\
&\preceq \text{rmax}\{A_I((f(z) * f(y)) * (f(z) * f(x))), A_I(f(y))\} && ((4.4.11))\\
&= \text{rmax}\{A_I(f(z \cdot y) * f(z \cdot x)), A_I(f(y))\}\\
&= \text{rmax}\{A_I(f((z \cdot y) \cdot (z \cdot x))), A_I(f(y))\}\\
&= \text{rmax}\{f^{-1}(A)_I((z \cdot y) \cdot (z \cdot x)), f^{-1}(A)_I(y)\}, && ((4.5.7))\\
f^{-1}(A)_F(x) &= A_F(f(x)) && ((4.5.7))\\
&\succeq \text{rmin}\{A_F((f(z) * f(y)) * (f(z) * f(x))), A_F(f(y))\} && ((4.4.11))\\
&= \text{rmin}\{A_F(f(z \cdot y) * f(z \cdot x)), A_F(f(y))\}\\
&= \text{rmin}\{A_F(f((z \cdot y) \cdot (z \cdot x))), A_F(f(y))\}\\
&= \text{rmin}\{f^{-1}(A)_F((z \cdot y) \cdot (z \cdot x)), f^{-1}(A)_F(y)\}, && ((4.5.7))\\
f^{-1}(\lambda)_T(x) &= \lambda_T(f(x)) && ((4.5.8))\\
&\leq \max\{\lambda_T((f(z) * f(y)) * (f(z) * f(x))), \lambda_T(f(y))\} && ((4.4.12))\\
&= \max\{\lambda_T(f(z \cdot y) * f(z \cdot x)), \lambda_T(f(y))\}\\
&= \max\{\lambda_T(f((z \cdot y) \cdot (z \cdot x))), \lambda_T(f(y))\}\\
&= \max\{f^{-1}(\lambda)_T((z \cdot y) \cdot (z \cdot x)), f^{-1}(\lambda)_T(y)\}, && ((4.5.8))\\
f^{-1}(\lambda)_I(x) &= \lambda_I(f(x)) && ((4.5.8))\\
&\geq \min\{\lambda_I((f(z) * f(y)) * (f(z) * f(x))), \lambda_I(f(y))\} && ((4.4.12))
\end{aligned}
$$

$$
\begin{aligned}
&= \min\{\lambda_I(f(z \cdot y) * f(z \cdot x)), \lambda_I(f(y))\} \\
&= \min\{\lambda_I(f((z \cdot y) \cdot (z \cdot x))), \lambda_I(f(y))\} \\
&= \min\{f^{-1}(\lambda)_I((z \cdot y) \cdot (z \cdot x)), f^{-1}(\lambda)_I(y)\}, && ((4.5.8)) \\
f^{-1}(\lambda)_F(x) &= \lambda_F(f(x)) && ((4.5.8)) \\
&\leq \max\{\lambda_F((f(z) * f(y)) * (f(z) * f(x))), \lambda_F(f(y))\} && ((4.4.12)) \\
&= \max\{\lambda_F(f(z \cdot y) * f(z \cdot x)), \lambda_F(f(y))\} \\
&= \max\{\lambda_F(f((z \cdot y) \cdot (z \cdot x))), \lambda_F(f(y))\} \\
&= \max\{f^{-1}(\lambda)_F((z \cdot y) \cdot (z \cdot x)), f^{-1}(\lambda)_F(y)\}. && ((4.5.8))
\end{aligned}
$$

Hence, $f^{-1}(\mathscr{A})$ is a neutrosophic cubic strong UP-ideal of X. □

Definition 4.5.8 A NCS $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ in X has *NCS-property* if for any nonempty subset S of X, there exist elements $\alpha_{T,I,F}, \beta_{T,I,F} \in S$ (instead of $\alpha_T, \alpha_I, \alpha_F, \beta_T, \beta_I, \beta_F \in S$) such that

$$
\begin{aligned}
A_T(\alpha_T) &= \text{rsup}_{s \in S}\{A_T(s)\}, \\
A_I(\alpha_I) &= \text{rinf}_{s \in S}\{A_I(s)\}, \\
A_F(\alpha_F) &= \text{rsup}_{s \in S}\{A_F(s)\}, \\
\lambda_T(\beta_T) &= \inf_{s \in S}\{\lambda_T(s)\}, \\
\lambda_I(\beta_I) &= \sup_{s \in S}\{\lambda_I(s)\}, \text{ and} \\
\lambda_F(\beta_F) &= \inf_{s \in S}\{\lambda_F(s)\}.
\end{aligned}
$$

Definition 4.5.9 Let X and Y be any two nonempty sets and let $f : X \to Y$ be any function. A NCS $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ in X is said to be *f-invariant* if

$$
(\forall x, y \in X)(f(x) = f(y) \Rightarrow A_{T,I,F}(x) = A_{T,I,F}(y), \lambda_{T,I,F}(x) = \lambda_{T,I,F}(y)). \tag{4.5.10}
$$

Lemma 4.5.10 *Let $(X, \cdot, 0_X)$ and $(Y, *, 0_Y)$ be UP-algebras and let $f \colon X \to Y$*

be a UP-epimorphism. Let $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ *be an* f*-invariant NCS in* X *with NCS-property. For any* $x, y \in Y$*, there exist elements* $\alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x)$ *and* $\beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$ *such that*

$$f(A)_T(x) = A_T(\alpha_T), f(A)_I(x) = A_I(\alpha_I), f(A)_F(x) = A_F(\alpha_F),$$
$$f(\lambda)_T(x) = \lambda_T(\gamma_T), f(\lambda)_I(x) = \lambda_I(\gamma_I), f(\lambda)_F(x) = \lambda_F(\gamma_F),$$
$$f(A)_T(y) = A_T(\beta_T), f(A)_I(y) = A_I(\beta_I), f(A)_F(y) = A_F(\beta_F),$$
$$f(\lambda)_T(y) = \lambda_T(\phi_T), f(\lambda)_I(y) = \lambda_I(\phi_I), f(\lambda)_F(y) = \lambda_F(\phi_F),$$
$$f(A)_T(x * y) = A_T(\alpha_T \cdot \beta_T), f(A)_I(x * y) = A_I(\alpha_I \cdot \beta_I),$$
$$f(A)_F(x * y) = A_F(\alpha_F \cdot \beta_F),$$
$$f(\lambda)_T(x * y) = \lambda_T(\gamma_T \cdot \phi_T), f(\lambda)_I(x * y) = \lambda_I(\gamma_I \cdot \phi_I),$$
$$f(\lambda)_F(x * y) = \lambda_F(\gamma_F \cdot \phi_F).$$

Proof. Let $x, y \in Y$. Since f is surjective, we have $f^{-1}(x), f^{-1}(y)$, and $f^{-1}(x \cdot y)$ are nonempty subsets of X. Since $\mathscr{A}$ has NCS-property, there exist elements $\alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x), \beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$, and $a_{T,I,F}, b_{T,I,F} \in f^{-1}(x * y)$ such that

$$f(A)_T(x) = \text{rsup}_{s \in f^{-1}(x)}\{A_T(s)\} = A_T(\alpha_T),$$
$$f(A)_I(x) = \text{rinf}_{s \in f^{-1}(x)}\{A_I(s)\} = A_I(\alpha_I),$$
$$f(A)_F(x) = \text{rsup}_{s \in f^{-1}(x)}\{A_F(s)\} = A_F(\alpha_F),$$
$$f(\lambda)_T(x) = \inf_{s \in f^{-1}(x)}\{\lambda_T(s)\} = \lambda_T(\gamma_T),$$
$$f(\lambda)_I(x) = \sup_{s \in f^{-1}(x)}\{\lambda_I(s)\} = \lambda_I(\gamma_I),$$
$$f(\lambda)_F(x) = \inf_{s \in f^{-1}(x)}\{\lambda_F(s)\} = \lambda_F(\gamma_F),$$
$$f(A)_T(y) = \text{rsup}_{s \in f^{-1}(y)}\{A_T(s)\} = A_T(\beta_T),$$
$$f(A)_I(y) = \text{rinf}_{s \in f^{-1}(y)}\{A_I(s)\} = A_I(\beta_I),$$
$$f(A)_F(y) = \text{rsup}_{s \in f^{-1}(y)}\{A_F(s)\} = A_F(\beta_F),$$

$$f(\lambda)_T(y) = \inf_{s\in f^{-1}(y)}\{\lambda_T(s)\} = \lambda_T(\phi_T),$$
$$f(\lambda)_I(y) = \sup_{s\in f^{-1}(y)}\{\lambda_I(s)\} = \lambda_I(\phi_I),$$
$$f(\lambda)_F(y) = \inf_{s\in f^{-1}(y)}\{\lambda_F(s)\} = \lambda_F(\phi_F),$$

and

$$f(A)_T(x*y) = \mathrm{rsup}_{s\in f^{-1}(x*y)}\{A_T(s)\} = A_T(a_T),$$
$$f(A)_I(x*y) = \mathrm{rinf}_{s\in f^{-1}(x*y)}\{A_I(s)\} = A_I(a_I),$$
$$f(A)_F(x*y) = \mathrm{rsup}_{s\in f^{-1}(x*y)}\{A_F(s)\} = A_F(a_F),$$
$$f(\lambda)_T(x*y) = \inf_{s\in f^{-1}(x*y)}\{\lambda_T(s)\} = \lambda_T(b_T),$$
$$f(\lambda)_I(x*y) = \sup_{s\in f^{-1}(x*y)}\{\lambda_I(s)\} = \lambda_I(b_I),$$
$$f(\lambda)_F(x*y) = \inf_{s\in f^{-1}(x*y)}\{\lambda_F(s)\} = \lambda_F(b_F).$$

Since

$$f(a_T) = x*y = f(\alpha_T)*f(\beta_T) = f(\alpha_T\cdot\beta_T),$$
$$f(a_I) = x*y = f(\alpha_I)*f(\beta_I) = f(\alpha_I\cdot\beta_I),$$
$$f(a_F) = x*y = f(\alpha_F)*f(\beta_F) = f(\alpha_F\cdot\beta_F),$$
$$f(b_T) = x*y = f(\gamma_T)*f(\phi_T) = f(\gamma_T\cdot\phi_T),$$
$$f(b_I) = x*y = f(\gamma_I)*f(\phi_I) = f(\gamma_I\cdot\phi_I),$$
$$f(b_F) = x*y = f(\gamma_F)*f(\phi_F) = f(\gamma_F\cdot\phi_F),$$

and $\mathscr{A}$ is f-invariant, we have

$$f(A)_T(x*y) = A_T(a_T) = A_T(\alpha_T\cdot\beta_T),$$
$$f(A)_I(x*y) = A_I(a_I) = A_I(\alpha_I\cdot\beta_I),$$
$$f(A)_F(x*y) = A_F(a_F) = A_F(\alpha_F\cdot\beta_F),$$

$$f(\lambda)_T(x * y) = \lambda_T(b_T) = \lambda_T(\gamma_T \cdot \phi_T)$$

$$f(\lambda)_I(x * y) = \lambda_I(b_I) = \lambda_I(\gamma_I \cdot \phi_I)$$

$$f(\lambda)_F(x * y) = \lambda_F(b_{TF}) = \lambda_F(\gamma_F \cdot \phi_F).$$

□

Theorem 4.5.11 *Let* $(X, \cdot, 0_X)$ *and* $(Y, *, 0_Y)$ *be UP-algebras,* $f\colon X \to Y$ *be a UP-epimorphism, and* $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ *be a NCS in* X*. Then the following statements hold:*

(1) *If* $\mathscr{A}$ *is an* f*-invariant neutrosophic cubic UP-subalgebra of* X *with NCS-property, then the image* $f(\mathscr{A})$ *of* $\mathscr{A}$ *under* f *is a neutrosophic cubic UP-subalgebra of* Y*.*

(2) *If* $\mathscr{A}$ *is an* f*-invariant neutrosophic cubic near UP-filter of* X *with NCS-property, then the image* $f(\mathscr{A})$ *of* $\mathscr{A}$ *under* f *is a neutrosophic cubic near UP-filter of* Y*.*

(3) *If* $\mathscr{A}$ *is an* f*-invariant neutrosophic cubic UP-filter of* X *with NCS-property, then the image* $f(\mathscr{A})$ *of* $\mathscr{A}$ *under* f *is a neutrosophic cubic UP-filter of* Y*.*

(4) *If* $\mathscr{A}$ *is an* f*-invariant neutrosophic cubic UP-ideal of* X *with NCS-property, then the image* $f(\mathscr{A})$ *of* $\mathscr{A}$ *under* f *is a neutrosophic cubic UP-ideal of* Y*.*

(5) *If* $\mathscr{A}$ *is an* f*-invariant neutrosophic cubic strong UP-ideal of* X *with NCS-property, then the image* $f(\mathscr{A})$ *of* $\mathscr{A}$ *under* f *is a neutrosophic cubic strong UP-ideal of* Y*.*

Proof. (1) Assume that $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ is an f-invariant neutrosophic cubic UP-subalgebra of X with NCS-property. Let $x, y \in Y$. Since f is surjective, we have $f^{-1}(x), f^{-1}(y)$, and $f^{-1}(x * y)$ are nonempty. By Lemma 4.5.10, there exist elements $\alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x)$ and $\beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$ such that

$$f(A)_T(x) = A_T(\alpha_T), f(A)_I(x) = A_I(\alpha_I), f(A)_F(x) = A_F(\alpha_F),$$
$$f(\lambda)_T(x) = \lambda_T(\gamma_T), f(\lambda)_I(x) = \lambda_I(\gamma_I), f(\lambda)_F(x) = \lambda_F(\gamma_F),$$
$$f(A)_T(y) = A_T(\beta_T), f(A)_I(y) = A_I(\beta_I), f(A)_F(y) = A_F(\beta_F),$$
$$f(\lambda)_T(y) = \lambda_T(\phi_T), f(\lambda)_I(y) = \lambda_I(\phi_I), f(\lambda)_F(y) = \lambda_F(\phi_F),$$
$$f(A)_T(x * y) = A_T(\alpha_T \cdot \beta_T), f(A)_I(x * y) = A_I(\alpha_I \cdot \beta_I),$$
$$f(A)_F(x * y) = A_F(\alpha_F \cdot \beta_F),$$
$$f(\lambda)_T(x * y) = \lambda_T(\gamma_T \cdot \phi_T), f(\lambda)_I(x * y) = \lambda_I(\gamma_I \cdot \phi_I),$$
$$f(\lambda)_F(x * y) = \lambda_F(\gamma_F \cdot \phi_F).$$

Then

$$\begin{aligned}
f(A)_T(x * y) &= A_T(\alpha_T \cdot \beta_T) \\
&\succeq \text{rmin}\{A_T(\alpha_T), A_T(\beta_T)\} && ((4.4.1)) \\
&= \text{rmin}\{f(A)_T(x), f(A)_T(y)\}, \\
f(A)_I(x * y) &= A_I(\alpha_I \cdot \beta_I) \\
&\preceq \text{rmax}\{A_I(\alpha_I), A_I(\beta_I)\} && ((4.4.1)) \\
&= \text{rmax}\{f(A)_I(x), f(A)_I(y)\}, \\
f(A)_F(x * y) &= A_F(\alpha_F \cdot \beta_F) \\
&\succeq \text{rmin}\{A_F(\alpha_F), A_F(\beta_F)\} && ((4.4.1)) \\
&= \text{rmin}\{f(A)_F(x), f(A)_F(y)\}, \\
f(\lambda)_T(x * y) &= \lambda_T(\gamma_T \cdot \phi_T) \\
&\leq \max\{\lambda_T(\gamma_T), \lambda_T(\phi_T)\} && ((4.4.2)) \\
&= \max\{f(\lambda)_T(x), f(\lambda)_T(y)\}, \\
f(\lambda)_I(x * y) &= \lambda_I(\gamma_I \cdot \phi_I)
\end{aligned}$$

$$
\begin{aligned}
&\geq \min\{\lambda_I(\gamma_I), \lambda_I(\phi_I)\} && ((4.4.2))\\
&= \min\{f(\lambda)_I(x), f(\lambda)_I(y)\},\\
f(\lambda)_F(x*y) &= \lambda_F(\gamma_F \cdot \phi_F)\\
&\leq \max\{\lambda_F(\gamma_F), \lambda_F(\phi_F)\} && ((4.4.2))\\
&= \max\{f(\lambda)_F(x), f(\lambda)_F(y)\}.
\end{aligned}
$$

Hence, $f(\mathscr{A})$ is a neutrosophic cubic UP-subalgebra of Y.

(2) Assume that $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ is an f-invariant neutrosophic cubic near UP-filter of X with NCS-property. By Theorem 3.0.8 (1), we have $0_X \in f^{-1}(0_Y)$ and so $f^{-1}(0_Y) \neq \emptyset$. Thus

$$
\begin{pmatrix}
f(A)_T(0_Y) = \text{rsup}_{s\in f^{-1}(0_Y)}\{A_T(s)\} \succeq A_T(0_X)\\
f(A)_I(0_Y) = \text{rinf}_{s\in f^{-1}(0_Y)}\{A_I(s)\} \preceq A_I(0_X)\\
f(A)_F(0_Y) = \text{rsup}_{s\in f^{-1}(0_Y)}\{A_F(s)\} \succeq A_F(0_X)\\
f(\lambda)_T(0_Y) = \inf_{s\in f^{-1}(0_Y)}\{\lambda_T(s)\} \leq \lambda_T(0_X)\\
f(\lambda)_I(0_Y) = \sup_{s\in f^{-1}(0_Y)}\{\lambda_I(s)\} \geq \lambda_I(0_X)\\
f(\lambda)_F(0_Y) = \inf_{s\in f^{-1}(0_Y)}\{\lambda_F(s)\} \leq \lambda_F(0_X)
\end{pmatrix}. \tag{4.5.11}
$$

Let $y \in Y$. Since f is surjective, we have $f^{-1}(y) \neq \emptyset$. By (4.4.3) and (4.4.4), we have $A_T(0_X) \succeq A_T(s), A_I(0_X) \preceq A_I(s), A_F(0_X) \succeq A_F(s), \lambda_T(0_X) \leq \lambda_T(s), \lambda_I(0_X) \geq \lambda_I(s), \lambda_F(0_X) \leq \lambda_F(s)$ for all $s \in f^{-1}(y)$. Then $A_T(0_X)$ is an upper bound of $\{A_T(s)\}_{s\in f^{-1}(y)}$, $A_I(0_X)$ is a lower bound of $\{A_I(s)\}_{s\in f^{-1}(y)}$, $A_F(0_X)$ is an upper bound of $\{A_F(s)\}_{s\in f^{-1}(y)}$, $\lambda_T(0_X)$ is a lower bound of $\{\lambda_T(s)\}_{s\in f^{-1}(y)}$, $\lambda_I(0_X)$ is an upper bound of $\{\lambda_I(s)\}_{s\in f^{-1}(y)}$, and $\lambda_F(0_X)$ is a lower bound of $\{\lambda_F(s)\}_{s\in f^{-1}(y)}$. By (4.5.11), we have

$$
\begin{aligned}
f(A)_T(0_Y) \succeq A_T(0_X) \succeq \text{rsup}_{s\in f^{-1}(y)}\{A_T(s)\} = f(A)_T(y),\\
f(A)_I(0_Y) \preceq A_I(0_X) \preceq \text{rinf}_{s\in f^{-1}(y)}\{A_I(s)\} = f(A)_I(y),
\end{aligned}
$$

$$f(A)_F(0_Y) \succeq A_F(0_X) \succeq \text{rsup}_{s\in f^{-1}(y)}\{A_F(s)\} = f(A)_F(y),$$

$$f(\lambda)_T(0_Y) \leq \lambda_T(0_X) \leq \inf_{s\in f^{-1}(y)}\{\lambda_T(s)\} = f(\lambda)_T(y),$$

$$f(\lambda)_I(0_Y) \geq \lambda_I(0_X) \geq \sup_{s\in f^{-1}(y)}\{\lambda_I(s)\} = f(\lambda)_I(y),$$

$$f(\lambda)_F(0_Y) \leq \lambda_F(0_X) \leq \inf_{s\in f^{-1}(y)}\{\lambda_F(s)\} = f(\lambda)_F(y).$$

Let $x, y \in Y$. By Lemma 4.5.10, there exist elements $\alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x)$ and $\beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$ such that

$$f(A)_T(x) = A_T(\alpha_T), f(A)_I(x) = A_I(\alpha_I), f(A)_F(x) = A_F(\alpha_F),$$

$$f(\lambda)_T(x) = \lambda_T(\gamma_T), f(\lambda)_I(x) = \lambda_I(\gamma_I), f(\lambda)_F(x) = \lambda_F(\gamma_F),$$

$$f(A)_T(y) = A_T(\beta_T), f(A)_I(y) = A_I(\beta_I), f(A)_F(y) = A_F(\beta_F),$$

$$f(\lambda)_T(y) = \lambda_T(\phi_T), f(\lambda)_I(y) = \lambda_I(\phi_I), f(\lambda)_F(y) = \lambda_F(\phi_F),$$

$$f(A)_T(x*y) = A_T(\alpha_T \cdot \beta_T), f(A)_I(x*y) = A_I(\alpha_I \cdot \beta_I),$$

$$f(A)_F(x*y) = A_F(\alpha_F \cdot \beta_F),$$

$$f(\lambda)_T(x*y) = \lambda_T(\gamma_T \cdot \phi_T), f(\lambda)_I(x*y) = \lambda_I(\gamma_I \cdot \phi_I),$$

$$f(\lambda)_F(x*y) = \lambda_F(\gamma_F \cdot \phi_F).$$

Then

$$\begin{aligned} f(A)_T(x*y) &= A_T(\alpha_T \cdot \beta_T) \\ &\succeq A_T(\beta_T) && ((4.4.5)) \\ &= f(A)_T(y), \\ f(A)_I(x*y) &= A_T(\alpha_I \cdot \beta_I) \\ &\preceq A_I(\beta_I) && ((4.4.5)) \\ &= f(A)_I(y), \\ f(A)_F(x*y) &= A_F(\alpha_F \cdot \beta_F) \end{aligned}$$

$$
\begin{aligned}
&\succeq A_F(\beta_F) && ((4.4.5))\\
&= f(A)_F(y),\\
f(\lambda)_T(x*y) &= \lambda_T(\gamma_T \cdot \phi_T)\\
&\leq \lambda_T(\phi_T) && ((4.4.6))\\
&= f(\lambda)_T(y),\\
f(\lambda)_I(x*y) &= \lambda_I(\gamma_I \cdot \phi_I)\\
&\geq \lambda_I(\phi_I) && ((4.4.6))\\
&= f(\lambda)_I(y),\\
f(\lambda)_F(x*y) &= \lambda_F(\gamma_F \cdot \phi_F)\\
&\leq \lambda_F(\phi_F) && ((4.4.6))\\
&= f(\lambda)_F(y).
\end{aligned}
$$

Hence, $f(\mathscr{A})$ is a neutrosophic cubic near UP-filter of Y.

(3) Assume that $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ is an f-invariant neutrosophic cubic UP-filter of X with NCS-property. Then $\mathscr{A}$ is a neutrosophic cubic near UP-filter of X. By the proof of (2), we have $f(\mathscr{A})$ satisfies the conditions (4.4.3) and (4.4.4). Let $x, y \in Y$. By Lemma 4.5.10, there exist elements $\alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x)$ and $\beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$ such that

$$
\begin{aligned}
f(A)_T(x) = A_T(\alpha_T), f(A)_I(x) = A_I(\alpha_I), f(A)_F(x) = A_F(\alpha_F),\\
f(\lambda)_T(x) = \lambda_T(\gamma_T), f(\lambda)_I(x) = \lambda_I(\gamma_I), f(\lambda)_F(x) = \lambda_F(\gamma_F),\\
f(A)_T(y) = A_T(\beta_T), f(A)_I(y) = A_I(\beta_I), f(A)_F(y) = A_F(\beta_F),\\
f(\lambda)_T(y) = \lambda_T(\phi_T), f(\lambda)_I(y) = \lambda_I(\phi_I), f(\lambda)_F(y) = \lambda_F(\phi_F),\\
f(A)_T(x*y) = A_T(\alpha_T \cdot \beta_T), f(A)_I(x*y) = A_I(\alpha_I \cdot \beta_I),\\
f(A)_F(x*y) = A_F(\alpha_F \cdot \beta_F),\\
f(\lambda)_T(x*y) = \lambda_T(\gamma_T \cdot \phi_T), f(\lambda)_I(x*y) = \lambda_I(\gamma_I \cdot \phi_I),
\end{aligned}
$$

$$f(\lambda)_F(x * y) = \lambda_F(\gamma_F \cdot \phi_F).$$

Then

$$\begin{aligned}
f(A)_T(y) &= A_T(\beta_T) \\
&\succeq \text{rmin}\{A_T(\alpha_T \cdot \beta_T), A_T(\alpha_T)\} && ((4.4.7)) \\
&= \text{rmin}\{f(A)_T(x * y), f(A)_T(x)\}, \\
f(A)_I(y) &= A_I(\beta_I) \\
&\preceq \text{rmax}\{A_I(\alpha_I \cdot \beta_I), A_I(\alpha_I)\} && ((4.4.7)) \\
&= \text{rmax}\{f(A)_I(x * y), f(A)_I(x)\}, \\
f(A)_F(y) &= A_F(\beta_F) \\
&\succeq \text{rmin}\{A_F(\alpha_F \cdot \beta_F), A_F(\alpha_F)\} && ((4.4.7)) \\
&= \text{rmin}\{f(A)_F(x * y), f(A)_F(x)\}, \\
f(\lambda)_T(y) &= \lambda_T(\phi_T) \\
&\leq \max\{\lambda_T(\gamma_T \cdot \phi_T), \lambda_T(\gamma_T)\} && ((4.4.8)) \\
&= \max\{f(\lambda)_T(x * y), f(\lambda)_T(x)\}, \\
f(\lambda)_I(y) &= \lambda_I(\phi_I) \\
&\geq \min\{\lambda_I(\gamma_I \cdot \phi_I), \lambda_I(\gamma_I)\} && ((4.4.8)) \\
&= \min\{f(\lambda)_I(x * y), f(\lambda)_I(x)\}, \\
f(\lambda)_F(y) &= \lambda_F(\phi_F) \\
&\leq \max\{\lambda_F(\gamma_F \cdot \phi_F), \lambda_F(\gamma_F)\} && ((4.4.8)) \\
&= \max\{f(\lambda)_F(x * y), f(\lambda)_F(x)\}.
\end{aligned}$$

Hence, $f(\mathscr{A})$ is a neutrosophic cubic UP-filter of Y.

(4) Assume that $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ is an f-invariant neutrosophic cubic UP-ideal of X with NCS-property. Then $\mathscr{A}$ is a neutrosophic cubic UP-filter of

X. By the proof of (3), we have $f(\mathscr{A})$ satisfies the conditions (4.4.3) and (4.4.4). Let $x, y, z \in Y$. By Lemma 4.5.10, there exist elements $\alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x)$, $\beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$ and $\psi_{T,I,F}, \omega_{T,I,F} \in f^{-1}(z)$ such that

$$f(A)_T(y) = A_T(\beta_T), f(A)_I(y) = A_I(\beta_I), f(A)_F(y) = A_F(\beta_F),$$
$$f(\lambda)_T(y) = \lambda_T(\phi_T), f(\lambda)_I(y) = \lambda_I(\phi_I), f(\lambda)_F(y) = \lambda_F(\phi_F),$$
$$f(A)_T(x * z) = A_T(\alpha_T \cdot \psi_T), f(A)_I(x * z) = A_I(\alpha_I \cdot \psi_I),$$
$$f(A)_F(x * z) = A_F(\alpha_F \cdot \psi_F),$$
$$f(\lambda)_T(x * z) = \lambda_T(\gamma_T \cdot \omega_T), f(\lambda)_I(x * z) = \lambda_I(\gamma_I \cdot \omega_I),$$
$$f(\lambda)_F(x * z) = \lambda_F(\gamma_F \cdot \omega_F),$$
$$f(A)_T(x * (y * z)) = A_T(\alpha_T \cdot (\beta_T \cdot \psi_T)), f(A)_I(x * (y * z)) = A_I(\alpha_I \cdot (\beta_I \cdot \psi_I)),$$
$$f(A)_F(x * (y * z)) = A_F(\alpha_F \cdot (\beta_F \cdot \psi_F)),$$
$$f(\lambda)_T(x * (y * z)) = \lambda_T(\gamma_T \cdot (\phi_T \cdot \omega_T)), f(\lambda)_I(x * (y * z)) = \lambda_I(\gamma_I \cdot (\phi_I \cdot \omega_I)),$$
$$f(\lambda)_F(x * (y * z)) = \lambda_F(\gamma_F \cdot (\phi_F \cdot \omega_F)).$$

Then

$$\begin{aligned}
f(A)_T(x * z) &= A_T(\alpha_T \cdot \psi_T) \\
&\succeq \operatorname{rmin}\{A_T(\alpha_T \cdot (\beta_T \cdot \psi_T)), A_T(\beta_T)\} && ((4.4.9)) \\
&= \operatorname{rmin}\{f(A)_T(x * (y * z)), f(A)_T(y)\}, \\
f(A)_I(x * z) &= A_I(\alpha_I \cdot \psi_I) \\
&\preceq \operatorname{rmax}\{A_I(\alpha_I \cdot (\beta_I \cdot \psi_I)), A_I(\beta_I)\} && ((4.4.9)) \\
&= \operatorname{rmax}\{f(A)_I(x * (y * z)), f(A)_I(y)\}, \\
f(A)_F(x * z) &= A_F(\alpha_F \cdot \psi_F) \\
&\succeq \operatorname{rmin}\{A_F(\alpha_F \cdot (\beta_F \cdot \psi_F)), A_F(\beta_F)\} && ((4.4.9)) \\
&= \operatorname{rmin}\{f(A)_F(x * (y * z)), f(A)_F(y)\},
\end{aligned}$$

$$
\begin{aligned}
f(\lambda)_T(x*z) &= \lambda_T(\gamma_T\cdot\omega_T)\\
&\leq \max\{\lambda_T(\gamma_T\cdot(\phi_T\cdot\omega_T)),\lambda_T(\phi_T)\} && ((4.4.10))\\
&= \max\{f(\lambda)_T(x*(y*z)), f(\lambda)_T(y)\},\\
f(\lambda)_I(x*z) &= \lambda_I(\gamma_I\cdot\omega_I)\\
&\geq \min\{\lambda_I(\gamma_I\cdot(\phi_I\cdot\omega_I)),\lambda_I(\phi_I)\} && ((4.4.10))\\
&= \min\{f(\lambda)_I(x*(y*z)), f(\lambda)_I(y)\},\\
f(\lambda)_F(x*z) &= \lambda_F(\gamma_F\cdot\omega_F)\\
&\leq \max\{\lambda_F(\gamma_F\cdot(\phi_F\cdot\omega_F)),\lambda_F(\phi_F)\} && ((4.4.10))\\
&= \max\{f(\lambda)_F(x*(y*z)), f(\lambda)_F(y)\}.
\end{aligned}
$$

Hence, $f(\mathscr{A})$ is a neutrosophic cubic UP-ideal of Y.

(5) Assume that $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ is an f-invariant neutrosophic cubic strong UP-ideal of X with NCS-property. Then $\mathscr{A}$ is a neutrosophic cubic UP-ideal of X. By the proof of (4), we have $f(\mathscr{A})$ satisfies the conditions (4.4.3) and (4.4.4). Let $x, y, z \in Y$. By Lemma 4.5.10, there exist elements $\alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x)$, $\beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$ and $\psi_{T,I,F}, \omega_{T,I,F} \in f^{-1}(z)$ such that

$$
\begin{aligned}
f(A)_T(x) = A_T(\alpha_T), f(A)_I(x) = A_I(\alpha_I), f(A)_F(x) = A_F(\alpha_F),\\
f(\lambda)_T(x) = \lambda_T(\gamma_T), f(\lambda)_I(x) = \lambda_I(\gamma_I), f(\lambda)_F(x) = \lambda_F(\gamma_F),\\
f(A)_T(y) = A_T(\beta_T), f(A)_I(y) = A_I(\beta_I), f(A)_F(y) = A_F(\beta_F),\\
f(\lambda)_T(y) = \lambda_T(\phi_T), f(\lambda)_I(y) = \lambda_I(\phi_I), f(\lambda)_F(y) = \lambda_F(\phi_F),\\
f(A)_T((z*y)*(z*x)) = A_T((\psi_T\cdot\beta_T)\cdot(\psi_T\cdot\alpha_T)),\\
f(A)_I((z*y)*(z*x)) = A_I((\psi_I\cdot\beta_I)\cdot(\psi_I\cdot\alpha_I)),\\
f(A)_F((z*y)*(z*x)) = A_F((\psi_F\cdot\beta_F)\cdot(\psi_F\cdot\alpha_F)),\\
f(\lambda)_T((z*y)*(z*x)) = \lambda_T((\omega_T\cdot\phi_T)\cdot(\omega_T\cdot\gamma_T)),\\
f(\lambda)_I((z*y)*(z*x)) = \lambda_I((\omega_I\cdot\phi_I)\cdot(\omega_I\cdot\gamma_I)),
\end{aligned}
$$

$$f(\lambda)_F((z*y)*(z*x)) = \lambda_F((\omega_F\cdot\phi_F)\cdot(\omega_F\cdot\gamma_F)).$$

Then

$$\begin{aligned}
f(A)_T(x) &= A_T(\alpha_T) \\
&\succeq \text{rmin}\{A_T((\psi_T\cdot\beta_T)\cdot(\psi_T\cdot\alpha_T)), A_T(\beta_T)\} && ((4.4.11)) \\
&= \text{rmin}\{f(A)_T((z*y)*(z*x)), f(A)_T(y)\}, \\
f(A)_I(x) &= A_I(\alpha_I) \\
&\preceq \text{rmax}\{A_I((\psi_I\cdot\beta_I)\cdot(\psi_I\cdot\alpha_I)), A_I(\beta_I)\} && ((4.4.11)) \\
&= \text{rmax}\{f(A)_I((z*y)*(z*x)), f(A)_I(y)\}, \\
f(A)_F(x) &= A_F(\alpha_F) \\
&\succeq \text{rmin}\{A_F((\psi_F\cdot\beta_F)\cdot(\psi_F\cdot\alpha_F)), A_F(\beta_F)\} && ((4.4.11)) \\
&= \text{rmin}\{f(A)_F((z*y)*(z*x)), f(A)_F(y)\}, \\
f(\lambda)_T(x) &= \lambda_T(\gamma_T) \\
&\leq \max\{\lambda_T((\omega_T\cdot\phi_T)\cdot(\omega_T\cdot\gamma_T)), \lambda_T(\phi_T)\} && ((4.4.12)) \\
&= \max\{f(\lambda)_T((z*y)*(z*x)), f(\lambda)_T(y)\}, \\
f(\lambda)_I(x) &= \lambda_I(\gamma_I) \\
&\geq \min\{\lambda_I((\omega_I\cdot\phi_I)\cdot(\omega_I\cdot\gamma_I)), \lambda_I(\phi_I)\} && ((4.4.12)) \\
&= \min\{f(\lambda)_I((z*y)*(z*x)), f(\lambda)_I(y)\}, \\
f(\lambda)_F(x) &= \lambda_F(\gamma_F) \\
&\leq \max\{\lambda_F((\omega_F\cdot\phi_F)\cdot(\omega_F\cdot\gamma_F)), \lambda_F(\phi_F)\} && ((4.4.12)) \\
&= \max\{f(\lambda)_F((z*y)*(z*x)), f(\lambda)_F(y)\}.
\end{aligned}$$

Hence, $f(\mathscr{A})$ is a neutrosophic cubic strong UP-ideal of Y. $\square$

CHAPTER V

CONCLUSIONS

From the study, we get the following results.

1. Every neutrosophic UP-subalgebra of X satisfies the conditions (4.1.4), (4.1.5), and (4.1.6).

2. A NS Λ in X is constant if and only if it is a neutrosophic strong UP-ideal of X.

3. If Λ is a neutrosophic UP-subalgebra of X satisfying the following condition:

$$(\forall x, y \in X)\left(x \cdot y \neq 0 \Rightarrow \begin{cases} \lambda_T(x) \geq \lambda_T(y) \\ \lambda_I(x) \leq \lambda_I(y) \\ \lambda_F(x) \geq \lambda_F(y) \end{cases}\right),$$

   then Λ is a neutrosophic near UP-filter of X.

4. If Λ is a neutrosophic near UP-filter of X satisfying the following condition:

$$\lambda_T = \lambda_I = \lambda_F,$$

   then Λ is a neutrosophic strong UP-ideal of X.

5. If Λ is a neutrosophic UP-filter of X satisfying the following condition:

$$(\forall x, y, z \in X)\begin{pmatrix} \lambda_T(y \cdot (x \cdot z)) = \lambda_T(x \cdot (y \cdot z)) \\ \lambda_I(y \cdot (x \cdot z)) = \lambda_I(x \cdot (y \cdot z)) \\ \lambda_F(y \cdot (x \cdot z)) = \lambda_F(x \cdot (y \cdot z)) \end{pmatrix},$$

   then Λ is a neutrosophic UP-ideal of X.

6. If Λ is a NS in X satisfying the following condition:

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} \lambda_T(z) \geq \min\{\lambda_T(x), \lambda_T(y)\} \\ \lambda_I(z) \leq \max\{\lambda_I(x), \lambda_I(y)\} \\ \lambda_F(z) \geq \min\{\lambda_F(x), \lambda_F(y)\} \end{cases}\right),$$

then Λ is a neutrosophic UP-subalgebra of X.

7. If Λ is a NS in X satisfying the following condition:

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} \lambda_T(y) \geq \min\{\lambda_T(z), \lambda_T(x)\} \\ \lambda_I(y) \leq \max\{\lambda_I(z), \lambda_I(x)\} \\ \lambda_F(y) \geq \min\{\lambda_F(z), \lambda_F(x)\} \end{cases}\right),$$

then Λ is a neutrosophic UP-filter of X.

8. If Λ is a NS in X satisfying the following condition:

$$(\forall a, x, y, z \in X)\left(a \leq x \cdot (y \cdot z) \Rightarrow \begin{cases} \lambda_T(x \cdot z) \geq \min\{\lambda_T(a), \lambda_T(y)\} \\ \lambda_I(x \cdot z) \leq \max\{\lambda_I(a), \lambda_I(y)\} \\ \lambda_F(x \cdot z) \geq \min\{\lambda_F(a), \lambda_F(y)\} \end{cases}\right),$$

then Λ is a neutrosophic UP-ideal of X.

9. A NS Λ in X satisfies the following condition:

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} \lambda_T(z) \geq \lambda_T(y) \\ \lambda_I(z) \leq \lambda_I(y) \\ \lambda_F(z) \geq \lambda_F(y) \end{cases}\right)$$

if and only if Λ is a neutrosophic strong UP-ideal of X.

10. If the constant 0 of X is in a nonempty subset G of X, then a NS $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ in X satisfies the conditions (4.1.4), (4.1.5), and (4.1.6).

11. If a NS $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ in X satisfies the condition (4.1.4) (resp., (4.1.5), (4.1.6)), then the constant 0 of X is in G.

12. A NS $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ in X is a neutrosophic UP-subalgebra (resp., neutrosophic near UP-filter, neutrosophic UP-filter, neutrosophic UP-ideal, neutrosophic strong UP-ideal) of X if and only if a nonempty subset G of X is a UP-subalgebra (resp., near UP-filter, UP-filter, UP-ideal, strong UP-ideal) of X.

13. A NS Λ in X is a neutrosophic UP-subalgebra (resp., neutrosophic near UP-filter, neutrosophic UP-filter, neutrosophic UP-ideal) of X if and only if for all $\alpha, \beta, \gamma \in [0, 1]$, the sets $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are either empty or UP-subalgebras (resp., near UP-filter, UP-filter, UP-ideal) of X.

14. A NS Λ in X is a neutrosophic strong UP-ideal of X if and only if the sets $E(\lambda_T; \lambda_T(0)), E(\lambda_I; \lambda_I(0))$, and $E(\lambda_F; \lambda_F(0))$ are strong UP-ideals of X.

15. Every special neutrosophic UP-subalgebra of X satisfies the conditions (4.2.4), (4.2.5), and (4.2.6).

16. A NS Λ in X is a neutrosophic UP-subalgebra (resp., neutrosophic near UP-filter, neutrosophic UP-filter, neutrosophic UP-ideal, neutrosophic strong UP-ideal) of X if and only if $\overline{\Lambda}$ is a special neutrosophic UP-subalgebra (resp., special neutrosophic near UP-filter, special neutrosophic UP-filter, special neutrosophic UP-ideal, special neutrosophic strong UP-ideal) of X.

17. A NS Λ in X is constant if and only if it is a special neutrosophic strong UP-ideal of X.

18. If Λ is a special neutrosophic UP-subalgebra of X satisfying the following condition:

$$(\forall x, y \in X)\left(x \cdot y \neq 0 \Rightarrow \begin{cases} \lambda_T(x) \leq \lambda_T(y) \\ \lambda_I(x) \geq \lambda_I(y) \\ \lambda_F(x) \leq \lambda_F(y) \end{cases}\right),$$

then Λ is a special neutrosophic near UP-filter of X.

19. If Λ is a special neutrosophic near UP-filter of X satisfying the following condition:

$$\lambda_T = \lambda_I = \lambda_F,$$

then Λ is a special neutrosophic strong UP-ideal of X.

20. If Λ is a special neutrosophic UP-filter of X satisfying the following condition:

$$(\forall x, y, z \in X)\begin{pmatrix} \lambda_T(y \cdot (x \cdot z)) = \lambda_T(x \cdot (y \cdot z)) \\ \lambda_I(y \cdot (x \cdot z)) = \lambda_I(x \cdot (y \cdot z)) \\ \lambda_F(y \cdot (x \cdot z)) = \lambda_F(x \cdot (y \cdot z)) \end{pmatrix},$$

then Λ is a special neutrosophic UP-ideal of X.

21. If Λ is a NS in X satisfying the following condition:

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} \lambda_T(z) \leq \max\{\lambda_T(x), \lambda_T(y)\} \\ \lambda_I(z) \geq \min\{\lambda_I(x), \lambda_I(y)\} \\ \lambda_F(z) \leq \max\{\lambda_F(x), \lambda_F(y)\} \end{cases}\right),$$

then Λ is a special neutrosophic UP-subalgebra of X.

22. If Λ is a NS in X satisfying the following condition:

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} \lambda_T(y) \leq \max\{\lambda_T(z), \lambda_T(x)\} \\ \lambda_I(y) \geq \min\{\lambda_I(z), \lambda_I(x)\} \\ \lambda_F(y) \leq \max\{\lambda_F(z), \lambda_F(x)\} \end{cases}\right),$$

then Λ is a special neutrosophic UP-filter of X.

23. If Λ is a NS in X satisfying the following condition:

$$(\forall a, x, y, z \in X)\left(a \leq x \cdot (y \cdot z) \Rightarrow \begin{cases} \lambda_T(x \cdot z) \leq \max\{\lambda_T(a), \lambda_T(y)\} \\ \lambda_I(x \cdot z) \geq \min\{\lambda_I(a), \lambda_I(y)\} \\ \lambda_F(x \cdot z) \leq \max\{\lambda_F(a), \lambda_F(y)\} \end{cases}\right),$$

then Λ is a special neutrosophic UP-ideal of X.

24. A NS Λ in X satisfies the following condition:

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} \lambda_T(z) \leq \lambda_T(y) \\ \lambda_I(z) \geq \lambda_I(y) \\ \lambda_F(z) \leq \lambda_F(y) \end{cases}\right)$$

if and only if Λ is a special neutrosophic near UP-filter of X.

25. Let $\alpha^+, \alpha^-, \beta^+, \beta^-, \gamma^+, \gamma^- \in [0, 1]$. Then the following statements hold:

(1) $\overline{\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]} = {}^G\Lambda[^{1-\alpha^+,1-\beta^-,1-\gamma^+}_{1-\alpha^-,1-\beta^+,1-\gamma^-}]$, and

(2) $\overline{{}^G\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]} = \Lambda^G[^{1-\alpha^-,1-\beta^+,1-\gamma^-}_{1-\alpha^+,1-\beta^-,1-\gamma^+}]$.

26. If the constant 0 of X is in a nonempty subset G of X, then a NS ${}^G\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ in X satisfies the conditions (4.2.4), (4.2.5), and (4.2.6).

27. If a NS ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ in X satisfies the condition (4.2.4) (resp., (4.2.5), (4.2.6)), then the constant 0 of X is in G.

28. A NS ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ in X is a special neutrosophic UP-subalgebra (resp., special neutrosophic near UP-filter, special neutrosophic UP-filter, special neutrosophic UP-ideal, special neutrosophic strong UP-ideal) of X if and only if a nonempty subset G of X is a UP-subalgebra (resp., near UP-filter, UP-filter, UP-ideal, strong UP-ideal) of X.

29. A NS Λ in X is a special neutrosophic UP-subalgebra (resp., special neutrosophic near UP-filter, special neutrosophic UP-filter, special neutrosophic UP-ideal) of X if and only if for all $\alpha, \beta, \gamma \in [0, 1]$, the sets $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are either empty or UP-subalgebras (resp., near UP-filter, UP-filter, UP-ideal) of X.

30. If $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra of X, then

$$(\forall x \in X)(A_T(0) \succeq A_T(x)),$$
$$(\forall x \in X)(A_I(0) \preceq A_I(x)),$$
$$(\forall x \in X)(A_F(0) \succeq A_F(x)).$$

31. An IVNS $\mathbf{A}$ in X is constant if and only if it is an interval-valued neutrosophic strong UP-ideal of X.

32. If $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra of X satisfying the following condition:

$$(\forall x, y \in X)\left(x \cdot y \neq 0 \Rightarrow \begin{cases} A_T(x) \succeq A_T(y) \\ A_I(x) \preceq A_I(y) \\ A_F(x) \succeq A_F(y) \end{cases} \right),$$

then **A** is an interval-valued neutrosophic near UP-filter of X.

33. If **A** is an interval-valued neutrosophic near UP-filter of X satisfying the following condition:

$$A_T = A_I = A_F,$$

then **A** is an interval-valued neutrosophic strong UP-ideal of X.

34. If **A** is an interval-valued neutrosophic UP-filter of X satisfying the following condition:

$$(\forall x, y, z \in X)\begin{pmatrix} A_T(y \cdot (x \cdot z)) = A_T(x \cdot (y \cdot z)) \\ A_I(y \cdot (x \cdot z)) = A_I(x \cdot (y \cdot z)) \\ A_F(y \cdot (x \cdot z)) = A_F(x \cdot (y \cdot z)) \end{pmatrix},$$

then **A** is an interval-valued neutrosophic UP-ideal of X.

35. If **A** is an IVNS in X satisfying the following condition:

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} A_T(z) \succeq \text{rmin}\{A_T(x), A_T(y)\} \\ A_I(z) \preceq \text{rmax}\{A_I(x), A_I(y)\} \\ A_F(z) \succeq \text{rmin}\{A_F(x), A_F(y)\} \end{cases} \right),$$

then **A** is an interval-valued neutrosophic UP-subalgebra of X.

36. If **A** is an IVNS in X satisfying the following condition:

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} A_T(y) \succeq \text{rmin}\{A_T(z), A_T(x)\} \\ A_I(y) \preceq \text{rmax}\{A_I(z), A_I(x)\} \\ A_F(y) \succeq \text{rmin}\{A_F(z), A_F(x)\} \end{cases} \right),$$

then **A** is an interval-valued neutrosophic UP-filter of X.

37. If $\mathbf{A}$ is an IVNS in X satisfying the following condition:

$$(\forall a, x, y, z \in X)\left(a \leq x \cdot (y \cdot z) \Rightarrow \begin{cases} A_T(x \cdot z) \succeq \text{rmin}\{A_T(a), A_T(y)\} \\ A_I(x \cdot z) \preceq \text{rmax}\{A_I(a), A_I(y)\} \\ A_F(x \cdot z) \succeq \text{rmin}\{A_F(a), A_F(y)\} \end{cases}\right),$$

then $\mathbf{A}$ is an interval-valued neutrosophic UP-ideal of X.

38. An IVNS A in X satisfies the following condition:

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} A_T(z) \succeq A_T(y) \\ A_I(z) \preceq A_I(y) \\ A_F(z) \succeq A_F(y) \end{cases}\right)$$

if and only if $\mathbf{A}$ is an interval-valued neutrosophic strong UP-ideal of X.

39. If the constant 0 of X is in a nonempty subset G of X, then the IVNS $\mathbf{A}^G[^{\tilde{a}^+, \tilde{b}^-, \tilde{c}^+}_{\tilde{a}^-, \tilde{b}^+, \tilde{c}^-}]$ in X satisfies the conditions (4.3.4), (4.3.5), and (4.3.6).

40. If the IVNS $\mathbf{A}^G[^{\tilde{a}^+, \tilde{b}^-, \tilde{c}^+}_{\tilde{a}^-, \tilde{b}^+, \tilde{c}^-}]$ in X satisfies the condition (4.3.4) (resp., (4.3.5), (4.3.6)), then the constant 0 of X is in G.

41. The IVNS $\mathbf{A}^G[^{\tilde{a}^+, \tilde{b}^-, \tilde{c}^+}_{\tilde{a}^-, \tilde{b}^+, \tilde{c}^-}]$ in X is an interval-valued neutrosophic UP-subalgebra (resp., interval-valued neutrosophic near UP-filter, interval-valued neutrosophic UP-filter, interval-valued neutrosophic UP-ideal, interval-valued neutrosophic strong UP-ideal) of X if and only if a nonempty subset G of X is a UP-subalgebra (resp., near UP-filters, UP-filters, UP-ideals, strong UP-ideal) of X.

42. An IVNS $\mathbf{A}$ in X is an interval-valued neutrosophic UP-subalgebra (resp., interval-valued neutrosophic near UP-filter, interval-valued neutrosophic UP-filter, interval-valued neutrosophic UP-ideal) of X if and only if for all

$\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$, the sets $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are either empty or UP-subalgebras (resp., near UP-filters, UP-filters, UP-ideals) of X.

43. An IVNS $\mathbf{A}$ in X is an interval-valued neutrosophic strong UP-ideal if and only if for all $\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$, the sets $E(A_T; A_T(0)), E(A_I; A_I(0))$, and $E(A_F; A_F(0))$ are strong UP-ideals of X.

44. If $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-subalgebra of X, then

$$(\forall x \in X)\begin{pmatrix} A_T(0) \succeq A_T(x) \\ A_I(0) \preceq A_I(x) \\ A_F(0) \succeq A_F(x) \end{pmatrix} \tag{P1}$$

and

$$(\forall x \in X)\begin{pmatrix} \lambda_T(0) \leq \lambda_T(x) \\ \lambda_I(0) \geq \lambda_I(x) \\ \lambda_F(0) \leq \lambda_F(x) \end{pmatrix}. \tag{P2}$$

45. A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X is a neutrosophic cubic UP-subalgebra (resp., neutrosophic cubic near UP-filter, neutrosophic cubic UP-filter, neutrosophic cubic UP-ideal, neutrosophic cubic strong UP-ideal) of X if and only if the IVNS $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra (resp., interval-valued neutrosophic near UP-filter, interval-valued neutrosophic UP-filter, interval-valued neutrosophic UP-ideal, interval-valued neutrosophic strong UP-ideal) of X and the NS Λ is a special neutrosophic UP-subalgebra (resp., special neutrosophic near UP-filter, special neutrosophic UP-filter, special neutrosophic UP-ideal, special neutrosophic strong UP-ideal) of X.

46. A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X is constant if and only if it is a neutrosophic cubic strong UP-ideal of X.

47. If $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-subalgebra of X satisfying the following condition:

$$(\forall x, y \in X)\left(x \cdot y \neq 0 \Rightarrow \begin{cases} A_T(x) \succeq A_T(y) \\ A_I(x) \preceq A_I(y) \\ A_F(x) \succeq A_F(y) \\ \lambda_T(x) \leq \lambda_T(y) \\ \lambda_I(x) \geq \lambda_I(y) \\ \lambda_F(x) \leq \lambda_F(y) \end{cases} \right),$$

then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic near UP-filter of X.

48. If $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic near UP-filter of X satisfying the following condition:

$$A_T = A_I = A_F, \lambda_T = \lambda_I = \lambda_F,$$

then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic strong UP-ideal of X.

49. If $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-filter of X satisfying the following condition:

$$(\forall x, y, z \in X)\begin{pmatrix} A_T(y \cdot (x \cdot z)) = A_T(x \cdot (y \cdot z)) \\ A_I(y \cdot (x \cdot z)) = A_I(x \cdot (y \cdot z)) \\ A_F(y \cdot (x \cdot z)) = A_F(x \cdot (y \cdot z)) \\ \lambda_T(y \cdot (x \cdot z)) = \lambda_T(x \cdot (y \cdot z)) \\ \lambda_I(y \cdot (x \cdot z)) = \lambda_I(x \cdot (y \cdot z)) \\ \lambda_F(y \cdot (x \cdot z)) = \lambda_F(x \cdot (y \cdot z)) \end{pmatrix},$$

then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-ideal of X.

50. If $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a NCS in X satisfying the following condition:

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} A_T(z) \succeq \text{rmin}\{A_T(x), A_T(y)\} \\ A_I(z) \preceq \text{rmax}\{A_I(x), A_I(y)\} \\ A_F(z) \succeq \text{rmin}\{A_F(x), A_F(y)\} \\ \lambda_T(z) \leq \max\{\lambda_T(x), \lambda_T(y)\} \\ \lambda_I(z) \geq \min\{\lambda_I(x), \lambda_I(y)\} \\ \lambda_F(z) \leq \max\{\lambda_F(x), \lambda_F(y)\} \end{cases} \right),$$

then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-subalgebra of X.

51. If $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a NCS in X satisfying the following condition:

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} A_T(y) \succeq \text{rmin}\{A_T(z), A_T(x)\} \\ A_I(y) \preceq \text{rmax}\{A_I(z), A_I(x)\} \\ A_F(y) \succeq \text{rmin}\{A_F(z), A_F(x)\} \\ \lambda_T(y) \leq \max\{\lambda_T(z), \lambda_T(x)\} \\ \lambda_I(y) \geq \min\{\lambda_I(z), \lambda_I(x)\} \\ \lambda_F(y) \leq \max\{\lambda_F(z), \lambda_F(x)\} \end{cases} \right),$$

then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-filter of X.

52. If $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a NCS in X satisfying the following condition:

$$(\forall a, x, y, z \in X)\left(a \leq x \cdot (y \cdot z) \Rightarrow \begin{cases} A_T(x \cdot z) \succeq \text{rmin}\{A_T(a), A_T(y)\} \\ A_I(x \cdot z) \preceq \text{rmax}\{A_I(a), A_I(y)\} \\ A_F(x \cdot z) \succeq \text{rmin}\{A_F(a), A_F(y)\} \\ \lambda_T(x \cdot z) \leq \max\{\lambda_T(a), \lambda_T(y)\} \\ \lambda_I(x \cdot z) \geq \min\{\lambda_I(a), \lambda_I(y)\} \\ \lambda_F(x \cdot z) \leq \max\{\lambda_F(a), \lambda_F(y)\} \end{cases} \right),$$

then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-ideal of X.

53. A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X satisfies the following condition:

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} A_T(z) \succeq A_T(y) \\ A_I(z) \preceq A_I(y) \\ A_F(z) \succeq A_F(y) \\ \lambda_T(z) \leq \lambda_T(y) \\ \lambda_I(z) \geq \lambda_I(y) \\ \lambda_F(z) \leq \lambda_F(y) \end{cases} \right)$$

if and only if $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic strong UP-ideal of X.

54. A NCS $\mathscr{A}^G[[^{\tilde{a}^+, \tilde{b}^-, \tilde{c}^+}_{\tilde{a}^-, \tilde{b}^+, \tilde{c}^-}], [^{\alpha^-, \beta^+, \gamma^-}_{\alpha^+, \beta^-, \gamma^+}]]$ in X is a neutrosophic cubic UP-subalgebra (resp., neutrosophic cubic near UP-filter, neutrosophic cubic UP-filter, neutrosophic cubic UP-ideal, neutrosophic cubic strong UP-ideal) of X if and only if a nonempty subset G of X is a UP-subalgebra (resp., near UP-filter, UP-filter, UP-ideal, strong UP-ideal) of X.

55. A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X is a neutrosophic cubic UP-subalgebra (resp., neu-

trosophic cubic near UP-filter, neutrosophic cubic UP-filter, neutrosophic cubic UP-ideal) of X if and only if for all $[s_{T_1}, s_{T_2}], [s_{I_1}, s_{I_2}], [s_{F_1}, s_{F_2}] \in [[0,1]]$ and $t_T, t_I, t_F \in [0,1]$, the sets $U(A_T; [s_{T_1}, s_{T_2}]), L(A_I; [s_{I_1}, s_{I_2}])$, $U(A_F; [s_{F_1}, s_{F_2}]), L(\lambda_T; t_T), U(\lambda_I; t_I)$, and $L(\lambda_F; t_F)$ are either empty or UP-subalgebras (resp., near UP-filter, UP-filter, UP-ideal) of X.

56. A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X is a neutrosophic cubic strong UP-ideal of X if and only if the sets $E(A_T; A_T(0)), E(A_I; A_I(0)), E(A_F; A_F(0)), E(\lambda_T, \lambda_T(0))$, $E(\lambda_I, \lambda_I(0))$, and $E(\lambda_F, \lambda_F(0))$ are strong UP-ideals of X.

57. Every neutrosophic cubic UP-filter (resp., neutrosophic cubic UP-ideal, neutrosophic cubic strong UP-ideal) of X is order preserving.

58. Let $(X, \cdot, 0_X)$ and $(Y, *, 0_Y)$ be UP-algebras, $f \colon X \to Y$ be a UP-homomorphism, and $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ be a NCS in Y. Then the following statements hold:

    (1) If $\mathscr{A}$ is a neutrosophic cubic UP-subalgebra of Y, then the inverse image $f^{-1}(\mathscr{A})$ of $\mathscr{A}$ under f is a neutrosophic cubic UP-subalgebra of X.

    (2) If $\mathscr{A}$ is a neutrosophic cubic near UP-filter of Y which is order preserving, then the inverse image $f^{-1}(\mathscr{A})$ of $\mathscr{A}$ under f is a neutrosophic cubic near UP-filter of X.

    (3) If $\mathscr{A}$ is a neutrosophic cubic UP-filter of Y, then the inverse image $f^{-1}(\mathscr{A})$ of $\mathscr{A}$ under f is a neutrosophic cubic UP-filter of X.

    (4) If $\mathscr{A}$ is a neutrosophic cubic UP-ideal of Y, then the inverse image $f^{-1}(\mathscr{A})$ of $\mathscr{A}$ under f is a neutrosophic cubic UP-ideal of X.

    (5) If $\mathscr{A}$ is a neutrosophic cubic strong UP-ideal of Y, then the inverse image $f^{-1}(\mathscr{A})$ of $\mathscr{A}$ under f is a neutrosophic cubic strong UP-ideal of X.

59. Let $(X, \cdot, 0_X)$ and $(Y, *, 0_Y)$ be UP-algebras and let $f\colon X \to Y$ be a UP-epimorphism. Let $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ be an f-invariant NCS in X with NCS-property. For any $x, y \in Y$, there exist elements $\alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x)$ and $\beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$ such that

$$\begin{aligned}
f(A)_T(x) = A_T(\alpha_T), f(A)_I(x) = A_I(\alpha_I), f(A)_F(x) = A_F(\alpha_F),\\
f(\lambda)_T(x) = \lambda_T(\gamma_T), f(\lambda)_I(x) = \lambda_I(\gamma_I), f(\lambda)_F(x) = \lambda_F(\gamma_F),\\
f(A)_T(y) = A_T(\beta_T), f(A)_I(y) = A_I(\beta_I), f(A)_F(y) = A_F(\beta_F),\\
f(\lambda)_T(y) = \lambda_T(\phi_T), f(\lambda)_I(y) = \lambda_I(\phi_I), f(\lambda)_F(y) = \lambda_F(\phi_F),\\
f(A)_T(x * y) = A_T(\alpha_T \cdot \beta_T), f(A)_I(x * y) = A_I(\alpha_I \cdot \beta_I),\\
f(A)_F(x * y) = A_F(\alpha_F \cdot \beta_F),\\
f(\lambda)_T(x * y) = \lambda_T(\gamma_T \cdot \phi_T), f(\lambda)_I(x * y) = \lambda_I(\gamma_I \cdot \phi_I),\\
f(\lambda)_F(x * y) = \lambda_F(\gamma_F \cdot \phi_F).
\end{aligned}$$

60. Let $(X, \cdot, 0_X)$ and $(Y, *, 0_Y)$ be UP-algebras, $f\colon X \to Y$ be a UP-epimorphism, and $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ be a NCS in X. Then the following statements hold:

    (1) If $\mathscr{A}$ is an f-invariant neutrosophic cubic UP-subalgebra of X with NCS-property, then the image $f(\mathscr{A})$ of $\mathscr{A}$ under f is a neutrosophic cubic UP-subalgebra of Y.

    (2) If $\mathscr{A}$ is an f-invariant neutrosophic cubic near UP-filter of X with NCS-property, then the image $f(\mathscr{A})$ of $\mathscr{A}$ under f is a neutrosophic cubic near UP-filter of Y.

    (3) If $\mathscr{A}$ is an f-invariant neutrosophic cubic UP-filter of X with NCS-property, then the image $f(\mathscr{A})$ of $\mathscr{A}$ under f is a neutrosophic cubic UP-filter of Y.

    (4) If $\mathscr{A}$ is an f-invariant neutrosophic cubic UP-ideal of X with NCS-

property, then the image $f(\mathscr{A})$ of $\mathscr{A}$ under f is a neutrosophic cubic UP-ideal of Y.

(5) If $\mathscr{A}$ is an f-invariant neutrosophic cubic strong UP-ideal of X with NCS-property, then the image $f(\mathscr{A})$ of $\mathscr{A}$ under f is a neutrosophic cubic strong UP-ideal of Y.

BIBLIOGRAPHY

BIBLIOGRAPHY

[1] Ansari, M. A., Haidar, A., and Koam, A. N. A. (2018). On a graph associated to UP-algebras. **Math. Comput. Appl.**, 23(4), 61.

[2] Ansari, M. A., Koam, A. N. A., and Haider, A. (2019). Rough set theory applied to UP-algebras. **Ital. J. Pure Appl. Math.**, 42, 388-402.

[3] Dokkhamdang, N., Kesorn, A., and Iampan, A. (2018). Generalized fuzzy sets in UP-algebras. **Ann. Fuzzy Math. Inform.**, 16(2), 171-190.

[4] Guntasow, T., Sajak, S., Jomkham, A., and Iampan, A. (2017). Fuzzy translations of a fuzzy set in UP-algebras. **J. Indones. Math. Soc.**, 23(2), 1-19.

[5] Iampan, A. Multipliers and near UP-filters of UP-algebras. **J. Discrete Math. Sci. Cryptography**, page to appear.

[6] Iampan, A. (2017). A new branch of the logical algebra: UP-algebras. **J. Algebra Relat. Top.**, 5(1), 35-54.

[7] Iampan, A. (2018). Introducing fully UP-semigroups. **Discuss. Math., Gen. Algebra Appl.**, 38(2), 297-306.

[8] Iampan, A. (2019). The UP-isomorphism theorems for UP-algebras. **Discuss. Math., Gen. Algebra Appl.**, 39(1), 113-123.

[9] Imai, Y. and Iséki, K. (1966). On axiom systems of propositional calculi XIV. **Proc. Japan Acad.**, 42, 19-22.

[10] Iqbal, R., Zafar, S., and Sardar, M. S. (2016). Neutrosophic cubic subalgebras and neutrosophic cubic closed ideals of B-algebras. **Neutrosophic Sets Syst.**, 14, 47-60.

[11] Iséki, K. (1966). An algebra related with a propositional calculus. **Proc. Japan Acad.**, 42(1), 26-29.

[12] Jun, Y. B., Jung, S. T., and Kim, M. S. (2011). Cubic subguoup. **Ann. Fuzzy Math. Inform.**, 2(1), 9-15.

[13] Jun, Y. B., Kim, C. S., and Yang, K. O. (2012). Cubic sets. **Ann. Fuzzy Math. Inform**, 4(1), 83-98.

[14] Jun, Y. B., Kim, S. J., and Smarandache, F. (2018). Interval neutrosophic sets with applications in BCK/BCI-algebra. **Axioms**, 7(2), 23-35.

[15] Jun, Y. B., Smarandache, F., and Bordbar, H. (2017). Neutrosophic $\mathcal{N}$-structures applied to BCK/BCI-algebras. **Inform.**, 8(4), 128.

[16] Jun, Y. B., Smarandache, F., and Kim, C. S. (2017). Neutrosophic cubic sets. **New Math. Nat. Comput.**, 13(1), 41-54.

[17] Kaijae, W., Poungsumpao, P., Arayarangsi, S., and Iampan, A. (2016). UP-algebras characterized by their anti-fuzzy UP-ideals and anti-fuzzy UP-subalgebras. **Ital. J. Pure Appl. Math.**, 36, 667-692.

[18] Kawila, K., Udomsetchai, C., and Iampan, A. (2018). Bipolar fuzzy UP-algebras. **Math. Comput. Appl.**, 23(4), 69.

[19] Keawrahun, S. and Leerawat, U. (2011). On isomorphisms of SU-algebras. **Sci. Magna**, 7(2), 39-44.

[20] Kesorn, B., Maimun, K., Ratbandan, W., and Iampan, A. (2015). Intuitionistic fuzzy sets in UP-algebras. **Ital. J. Pure Appl. Math.**, 34, 339-364.

[21] Khan, M., Anis, S., Smarandache, F., and Jun, Y. B. (2017). Neutrosophic $\mathcal{N}$-structures and their applications in semigroups. **Ann. Fuzzy Math. Inform.**, 14(6), 583-598.

[22] Mordeson, J. N., Malik, D. S., and Kuroki, N. (2012). Fuzzy Semigroups, volume 131. **Springer**.

[23] Muhiuddin, G. (2018). Neutrosophic subsemigroups. **Ann. Commun. Math.**, 1(1), 1-10.

[24] Muhiuddin, G., Al-Kenani, A. N., Roh, E. H., and Jun, Y. B. (2019). Implicative neutrosophic quadruple BCK-algebras and ideals. **Symmetry**, 11(2), 277.

[25] Muhiuddin, G., Bordbar, H., Smarandache, F., and Jun, Y. B. (2018). Further results on $(\in, \in)$-neutrosophic subalgebras and ideals in BCK/BCI-algebras. **Neutrosophic Sets Syst.**, 20, 36-43.

[26] Muhiuddin, G. and Jun, Y. B. (2018). p-semisimple neutrosophic quadruple BCI-algebras and neutrosophic quadruple p-ideals. **Ann. Commun. Math.**, 1(1), 26-37.

[27] Muhiuddin, G., Kim, S. J., and Jun, Y. B. (2019). Implicative $\mathcal{N}$-ideals of BCK-algebras based on neutrosophic $\mathcal{N}$-structures. **Discrete Math. Algorithms Appl.**, 11(1), 1950011.

[28] Muhiuddin, G., Smarandache, F., and Jun, Y. B. (2019). Neutrosophic quadruple ideals in neutrosophic quadruple BCI-algebras. **Neutrosophic Sets Syst.**, 25, 161-173.

[29] Neggers, J. and Kim, H. S. (2002). On B-algebras. **Mat. Vesnik**, 54, 21-29.

[30] Prabpayak, C. and Leerawat, U. (2009). On ideals and congruences in KU-algebras. **Sci. Magna**, 5(1), 54-57.

[31] Rangsuk, P., Huana, P., and Iampan, A. (2019). Neutrosophic $\mathcal{N}$-structures over UP-algebras. **Neutrosophic Sets Syst.**, 28, 87-127.

[32] Satirad, A., Mosrijai, P., and Iampan, A. (2019). Formulas for finding UP-algebras. **Int. J. Math. Comput. Sci.**, 14(2), 403-409.

[33] Satirad, A., Mosrijai, P., and Iampan, A. (2019). Generalized power UP-algebras. **Int. J. Math. Comput. Sci.**, 14(1), 17-25.

[34] Senapati, T., Jun, Y. B., and Shum, K. P. (2018). Cubic set structure applied in UP-algebras. **Discrete Math. Algorithms Appl.**, 10(4), 1850049.

[35] Senapati, T., Kim, C. S., Bhowmik, M., and Pal, M. (2015). Cubic subalgebras and cubic closed ideals of B-algebras. **Fuzzy Inf. Eng.**, 7(2), 129-149.

[36] Senapati, T., Muhiuddin, G., and Shum, K. P. (2017). Representation of UP-algebras in interval-valued intuitionistic fuzzy environment. **Ital. J. Pure Appl. Math.**, 38, 497-517.

[37] Smarandache, F. (1999). A Unifying Field in Logic: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability. **American Research Press**, Rehoboth, NM.

[38] Somjanta, J., Thuekaew, N., Kumpeangkeaw, P., and Iampan, A. (2016). Fuzzy sets in UP-algebras. **Ann. Fuzzy Math. Inform.**, 12(6), 739-756.

[39] Songsaeng, M. and Iampan, A. (2018). $\mathcal{N}$-fuzzy UP-algebras and its level subsets. **J. Algebra Relat. Top.**, 6(1), 1- 24.

[40] Songsaeng, M. and Iampan, A. (2019). Fuzzy proper UP-filters of UP-algebras. **Honam Math. J.**, 41(3), 515-530.

[41] Sripaeng, S., Tanamoon, K., and Iampan, A. (2018). On anti Q-fuzzy UP-ideals and anti Q-fuzzy UP-subalgebras of UP-algebras. **J. Inf. Optim. Sci.**, 39(5), 1095-1127.

[42] Taboon, K., Butsri, P., and Iampan, A. (2020). A cubic set theory approach to UP-algebras. Manuscript accepted for publication in J. Interdiscip. Math.

[43] Tanamoon, K., Sripaeng, S., and Iampan, A. (2018). Q-fuzzy sets in UP-algebras. **Songklanakarin J. Sci. Technol.**, 40(1), 9- 29.

[44] Udten, N., Songseang, N., and Iampan, A. (2019). Translation and density of a bipolar-valued fuzzy set in UP-algebras. **Ital. J. Pure Appl. Math.**, 41, 469-496.

[45] Wang, H., Smarandache, F., Zhang, Y. Q., and Sunderraman, R. (2005). Interval Neutrosophic Sets and logic: Theory and Applications in Computing. **Hexis**, Phoenix, Ariz, USA.

[46] Zadeh, L. A. (1965). Fuzzy sets. **Inf. Cont.**, 8, 338-353.

[47] Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning-I. **Inf. Sci.**, 8, 199-249.

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Publications

Articles

1. **Songsaeng, M.** and Iampan, A. (2020). A novel approach to neutrosophic sets in UP-algebras. J. Math. Comput. Sci., 21(1): 78-98

2. **Songsaeng, M.** and Iampan, A. (2020). Neutrosophic sets in UP-algebras by means of interval-valued fuzzy sets. J. Int. Math. Virtual Inst., 10(1): 93-122.

3. **Songsaeng, M.** and Iampan, A. (2019). Neutrosophic set theory applied to UP-algebras. Eur. J. Pure Appl. Math., 12(4): 1382-1409.

In process

1. **Songsaeng, M.** and Iampan, A. (2020). (Submitted). Neutrosophic cubic set theory applied to UP-algebras.

2. **Songsaeng, M.** and Iampan, A. (2020). (Submitted). Image and inverse image of neutrosophic cubic sets in UP-algebras under UP-homomorphisms.

Others

1. Iampan, A., **Songsaeng, M.**, and Muhiuddin G. (2020). (Submitted). Fuzzy duplex UP-algebras.

2. Iampan, A., Satirad, A., and **Songsaeng, M.** (2020). (Accepted). A note on UP-hyperalgebras. J. Algebr. Hyperstruct. Log. Algebr.

3. **Songsaeng, M.** and Iampan, A. (2019). Fuzzy proper UP-filters of UP-algebras. Honam Math. J., 41(3): 515-530.

4. **Songsaeng, M.** and Iampan, A. (2018). $\mathcal{N}$-fuzzy UP-algebras and its level subsets. J. Algebra Relat. Top., 6(1): 1-24.

Conference presentations

1. **Songsaeng, M.** (May 23-24, 2019). Neutrosophic cubic set theory applied to UP-algebras. In The 11th National Science Research Conference. Srinakharinwirot University, Bangkok, Thailand.